

THE MILLMAN EFFECT IN CESIUM  
BEAM ATOMIC FREQUENCY STANDARDS

A.G. Mungall  
National Research Council, Ottawa, Canada

ABSTRACT

A systematic frequency shift which reverses sign with either C field or beam reversal and which can be interpreted as the consequence of the Millman effect has been found in the new NRC cesium beam clock, CsV. A method of calculating the resonance asymmetry resulting from this effect and the dependence on the velocity distribution and exciting microwave power level of the frequency shift caused by it is described. Application of this theory to CsV leads to reasonable agreement with experimental results. Possible origins for the Millman effect in cesium beam standards are discussed, and it is concluded that the most likely source is a very small variation in the direction of the C field over normal beam trajectories within the microwave interaction regions.

I. INTRODUCTION

Cesium beam frequency standards, first developed in the mid-1950's, have undergone continuous improvement in accuracy<sup>1</sup> from an initial value of the order of  $10^{-9}$  to current values approaching  $10^{-13}$ . One of the results of such improvements in performance is that certain systematic errors which were at one time of minor consequence have now become of such importance that they can constitute the primary factors limiting the stability or the accuracy of the standard.

During the 20-year development period of these standards, the limiting instabilities and inaccuracies were to a large extent related to factors such as magnetic field inhomogeneities, crystal oscillator instability, indeterminate cavity phase differences, unknown atomic beam velocity distributions, and spectral impurities in the microwave excitation signal. Within the past five years, several methods have been described which provide estimates of the actual velocity distributions existing in practical cesium standards, and these have led to much improved understanding and measurement of the power dependent frequency shifts arising from the cavity phase difference and the second order Doppler effect. Advances in crystal oscillator design have also led to the essential elimination of the more or less unpredictable power-dependent frequency shifts due to spectral impurities.

However, as in other areas of high precision measurements, the

consecutive removal of each limiting source of systematic error has tended to expose more clearly further underlying sources, and recent work on CsV at NRC has proven no exception to this rule. A previous paper<sup>2</sup> on CsV outlined a number of sources of error, and at that time (1973) the limiting factor was spectral impurities in the exciting microwave signal which caused unpredictable time-dependent frequency shifts which varied with the power level. Extensive redesign and reconstruction of the electronics systems led to the final apparent elimination of these difficulties, and also to the measurement of another important frequency shift. This shift, which reverses sign with C field or beam reversal, can be interpreted as the consequence of the Millman effect, whose presence in CsV had not been previously detected. Resolution of this problem was accomplished during April, 1975, and since May 1, 1975, CsV has operated satisfactorily as a clock.

This paper will discuss the experimental determination of the Millman effect in CsV, propose a possible explanation for its origin in this standard, and also outline a general method of calculating its dependence on transition excitation level, beam velocity distribution, and servo-system modulation offset frequency.

## II. THEORY

### A. Theory of the Millman Effect

As outlined by Ramsey<sup>3</sup>, and first described by Millman<sup>4</sup>, the Millman effect is an alteration in the frequency experienced by an atom in an oscillating exciting field resulting from a change in direction of that field relative to a d.c. orienting magnetic field during its passage through the excitation region. It is in effect a spatially-induced frequency increment to the impressed oscillating field. Although this effect was originally considered to act only when the oscillating and orienting fields are perpendicular ( $\pi$  transitions), Hahn<sup>7</sup> has shown that a similar effect also occurs when they are parallel ( $\sigma$  transitions) if a spatial rotation of the orienting field occurs along the direction of the beam. Since cesium beam atomic frequency standards are based on the  $\sigma$  transition, ( $F = 4, m_F = 0$ )  $\leftrightarrow$  ( $F = 3, m_F = 0$ ), this effect is of importance in such standards. As pointed out by Hahn, it is dependent on the directions of both the orienting field and the beam. It is thus experimentally separable from the cavity phase difference frequency shift since the latter is dependent only on the beam direction and not on the direction of the orienting field.

The magnitude of the Millman effect, expressed as an incremental angular frequency  $\delta\omega_m$ , is related to the change in angle between

the directing and oscillating fields,  $\gamma$ , during the passage of an atom travelling at a velocity  $v$  through an interaction region of length  $\ell$  by

$$\delta\omega_m = \frac{\gamma v}{\ell} \quad (1)$$

#### B. Calculation of the Frequency Shift Averaged Over a Distribution of Atomic Velocities.

The dependence of  $\delta\omega_m$  on  $v$  indicates that in the general case for which a range of atomic velocities occurs the Millman effect not only causes a frequency shift but also gives rise to a resonance asymmetry, with consequent dependence of the measured transition frequency on exciting power level, velocity distribution, and servo-system modulation frequency offset. The form of its dependence can be expected to be quite different from either that arising from the second order Doppler effect, which varies as  $v^2/2c^2$ , or that for the cavity phase difference, which causes only a phase change between the two oscillating fields. Since in practical cesium standards the atomic beam includes atoms having a range of velocities comprising an appreciable fraction of the Maxwellian distribution, some method of integrating equation 1 over such a fraction is essential in the explanation of the measured frequency shifts.

A method first employed at NRC for calculating the frequency shift arising from the second order Doppler effect<sup>5</sup> and later for that due to a cavity phase difference<sup>6</sup>, can also be employed in the present case. It involves the assumption that the distribution function of the velocities of the atoms contributing to the resonance can be satisfactorily approximated by a truncated Maxwellian distribution, with lower and upper velocity cut-offs determined by the physical constants of the standard in question. In accordance with the terminology used previously, an atomic velocity  $v_i$  is related to the most probable velocity in the cesium oven,  $\alpha$ , by

$$P_i = v_i/\alpha \quad (2)$$

and the maximum and minimum velocities by

$$P_{max} = v_{max}/\alpha \quad \text{and} \quad P_{min} = v_{min}/\alpha \quad (3)$$

The transition probability  $P_i$  for a velocity defined by  $p_i$  is then calculated from Ramsey's general expression<sup>5</sup>, with  $\omega$  replaced by  $\omega - \delta\omega_m$ . In such a substitution, the value of  $\gamma$ , effectively a phase increment in each of the two oscillating fields, may be considered to a first approximation as  $\frac{1}{2} \cdot (L/\ell) \cdot \ell \delta\omega_m/v$ , by analogy with Ramsey's result that if  $\Delta\omega$  is the difference between the resonant frequency in the oscillatory and drift regions, the observed resonance is shifted by only  $(\ell/L) \cdot \Delta\omega$ .

$$P_i = 4 \sin^2 \theta \sin^2 \frac{a\ell}{2p_i\alpha} \cos \left[ \frac{(\omega_0 - \omega + \delta\omega_m)\ell}{2p_i\alpha} \cos \frac{a\ell}{2p_i\alpha} - \cos \theta \sin \frac{(\omega_0 - \omega + \delta\omega_m)}{2p_i\alpha} \sin \frac{a\ell}{2p_i\alpha} \right]^2 \quad (4)$$

where  $\sin \theta = -2b/a$ ,  $\cos \theta = (\omega_0 - \omega + \delta\omega_m)/a$ ,

$$\text{and } a = \left[ (\omega_0 - \omega + \delta\omega_m)^2 + (2b)^2 \right]^{1/2} \quad (5)$$

A weighted total transition probability  $P_t$ , defined by the upper and lower velocity limits, is then given by

$$P_t = \frac{\sum_i P_i w_i}{\sum_i w_i} \quad (6)$$

where the weights  $w_i$  are given by the truncated Maxwellian velocity distribution

$$w_i = p_i^3 \exp(1 - p_i^2) \text{ for } p_{\min} \leq p_i \leq p_{\max} \quad (7)$$

$$\text{and } w_i = 0 \text{ for } p_i < p_{\min} \text{ and } p_i > p_{\max} \quad (8)$$

The value of  $\alpha$  is known from the operating temperature of the cesium oven, and the value of  $b$  is determined from an approximate knowledge of the velocity limits and the dependence of the calculated and experimental maximum transition probabilities on exciting power level. The exact values of  $p_{\min}$  and  $p_{\max}$ , and hence the velocity distribution, are determined by curve fitting of calculated and experimental Ramsey resonance patterns. In practice, it is evident that sharp velocity cutoffs do not in fact occur, but their equivalent values can be estimated by this curve-fitting procedure to a precision of about  $\pm 0.05 \alpha$ .

The magnitude of the Millman effect frequency shift is then calculated by computer simulation of the response to the square wave frequency modulation applied to the atomic resonance by the clock servo system. Because of the resonance asymmetry the shift must be calculated for a range of frequency offsets. A transition probability  $P_{t1}$  is first calculated for an input frequency offset  $\Delta f_1$  below the resonant frequency, and then the corresponding value  $\Delta f_2$  for an equal

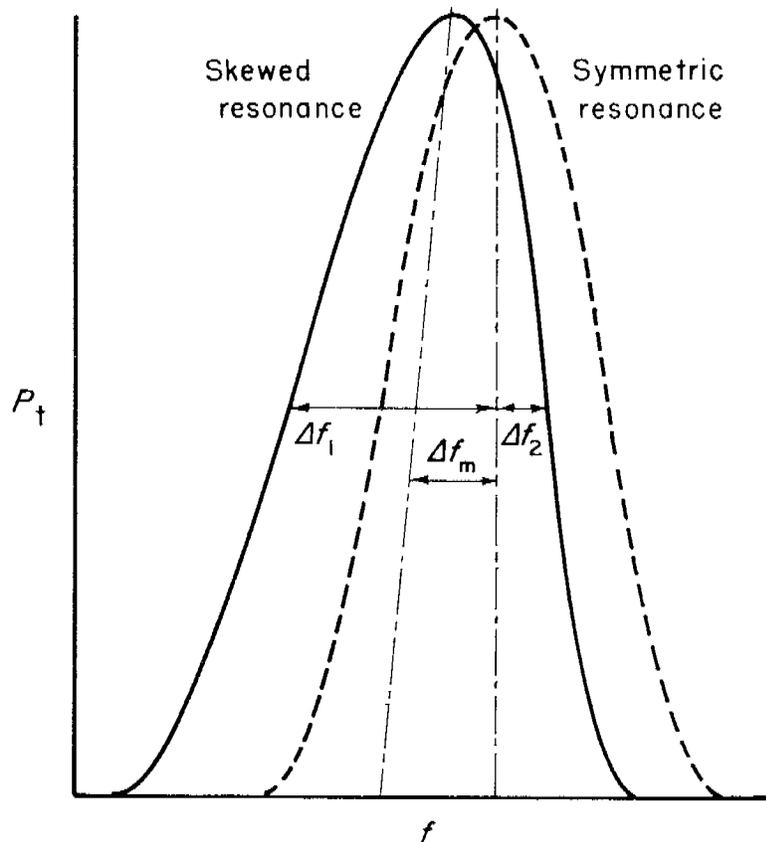


Figure 1. Calculation of the Millman effect frequency shift by computer simulation of servo-system interrogation of an asymmetric, frequency shifted resonance curve.

total transition probability  $P_{t2}$  above the resonant frequency is determined. As shown in figure 1, the Millman frequency shift  $\Delta f_m$  is then given by

$$\Delta f_m = \frac{\Delta f_2 - \Delta f_1}{2} \quad (9)$$

Inherent in this method is the assumption that the values of  $P_{t1}$  and  $P_{t2}$  are not affected by the rate of frequency switching between the upper and lower offset frequencies. In practice, no significant measurable change in the frequency of CsV occurs for switching

frequencies in the range 0.08 Hz to 1.0 Hz. The normal switching frequency used is 0.08 Hz.

### C. Origin of the Millman Effect in Cesium Beam Frequency Standards

It is apparent from the foregoing analysis that for parameters typical of present long beam laboratory cesium standards such as CsV, e.g., a beam velocity of about  $3 \times 10^4$  cm s<sup>-1</sup>, and drift and interaction lengths of about 200 cm and 1 cm, a relative rotation of the angle between orienting and oscillating fields in each interaction region of only  $10^{-4}$  radian will lead to a Millman effect frequency shift of about  $4 \times 10^{-13}$  or 0.004 Hz. This constitutes a surprisingly large frequency shift for a very small perturbation.

The origin of such a perturbation is open to some question, since present mechanical construction techniques do not provide precisions of this order. However, it should be noted that  $\gamma$  is actually a change in orientation rather than an absolute alignment error. Several possible explanations can be considered. Historically, the Millman effect was first noticed when current-carrying hairpins, curved upward at each end of the interaction region, were used to provide a vertical oscillating field perpendicular to the beam direction. The change in direction of the oscillating field magnetic vector over the curved sections of the hairpin was found to be the source of the effect. It is possible to propose a similar explanation in the present case, since an atom passing through the ends of the waveguide resonant cavity traverses regions of fringing electromagnetic fields which in general are not parallel with the fields within the cavity. These regions are, however, very short, and the field intensity in them is very low. In CsV, sections of K-band waveguide 2.5 cm in length cover the beam entrance and exit slots in the cavity, and effectively prevent radiation of microwave energy from the cavity. In these sections, the magnetic field vector is rotated through 90° compared with that in the cavity, and the attenuation of the microwave signal is such that the amplitude drops to about 1/3 in a distance of 5 mm. In addition, the efficiency of mode conversion from one waveguide to the other, and the radiation efficiency of the slot are both extremely low. The atomic transition probability for fields in these K-band sections would therefore be expected to be very small. If it is assumed, however, that a 90° angular change in the oscillating field does occur over a distance of 5 mm at both entrance and exit to the cavity, then the corresponding Millman effect is about 5 kHz. It is conceivable that a few atoms do in fact undergo such transitions, but it is apparent that such an explanation for the observed Millman effect, described later in this paper as about 0.002 Hz, cannot be valid.

Another possible explanation concerns misalignment of the wave-

guide in such a way that the oscillating magnetic field is not parallel with the orienting field. Such misalignment could produce the  $\pi$  transitions mentioned in the earlier paper<sup>2</sup>, but not a Millman effect which depends on a relative spatial rotation between oscillating and orienting fields in the beam direction.

A much more likely but unproven explanation can be based on the following reasoning. First, because of the beam optics design of CsV, which employs lateral deflection of a ribbon cross-section atomic beam by state selector magnets and passage of this beam through a central slit located on the oven-detector axis, the atomic trajectories through the waveguide interaction regions do not remain parallel with the longitudinal axis. Second, the orienting C field, produced by a four-rod current carrying structure enclosed by an open-ended cylindrical magnetic shield pierced by slots to permit entry of the waveguide cavity, is subject to small changes in direction with distance from the axis. The combination of these two physical circumstances leads to a slow relative rotation of the orienting C field with respect to the oscillating field experienced by the atom in its passage through the cavity. Furthermore, the level of microwave excitation within the cavity is usually one which approximates that for maximum transition probability. Consequently, both the necessary criteria for a Millman effect of the required order of magnitude are present. The experimental and semi-quantitative aspects of such an explanation will be examined in detail in the following section.

### III. EXPERIMENTAL

In the first preliminary evaluation of CsV, a search was made for the Millman effect, but none was found within the uncertainty of the measurements. This uncertainty was dictated by two factors: instabilities in the hydrogen masers used as reference standards, and instabilities in CsV arising most probably from time-dependent changes in the spectral purity of the exciting microwave signal. Subsequent to these preliminary tests, automatic tuning devices were built for the hydrogen masers, and extensive alterations were made to the electronics systems of CsV. In addition, a small change in the alignment of CsV was made when both cesium ovens were refilled at the end of 1974, in preparation for long-term operation of the device as a primary clock. As a consequence of this change of alignment, the Ramsey resonance patterns for the two beam directions are now approximately the same, as indicated in figure 2. If the second explanation proposed earlier is in fact valid, such a change in alignment would be expected to change the Millman effect, and in the experiments to be described later, a Millman effect was found. The more precise determination of this effect was facilitated by the significant improvements in frequency stability produced both in the masers and CsV. The relative stability

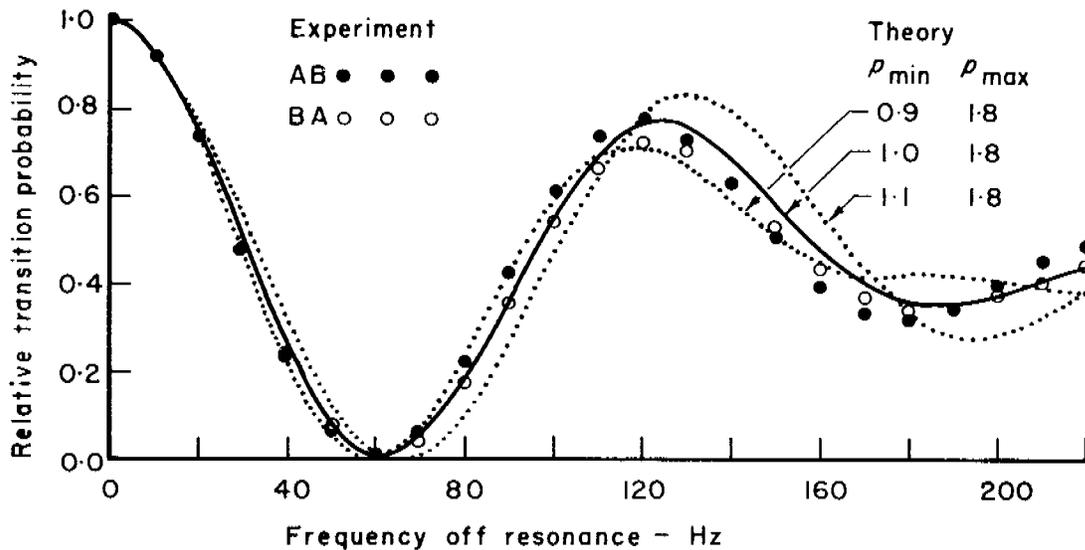


Figure 2. Normalized Ramsey resonance curves calculated from the theory for  $p_{max} = 1.8$  and  $p_{min} = 0.9, 1.0,$  and  $1.1$  compared with experimental results for beam directions AB and BA.

of the two masers was about  $2 \times 10^{-14}$  over periods of several hundred seconds to several days, and it was therefore possible to attain measurement uncertainties for systematic errors in CsV of between 1 and  $2 \times 10^{-14}$  for measurement periods in excess of about 30 minutes and up to several days.

As mentioned earlier, the Millman effect frequency shift is separable from that due to a cavity phase difference, since the latter depends only on beam direction whereas the former depends on both beam and C field direction. In order to measure the Millman effect frequency shift, measurements were made of the frequency of CsV relative to one of the hydrogen masers at several different power levels for both directions of the beam and the C field. A total of 4 beam and 12 C field reversals was made during April, 1975, and one beam reversal and 12 C field reversals during September, 1975. Some maser frequency drift, corroborated by simultaneous maser-maser frequency comparisons, occurred, and this was taken into account in the calculations of the frequency shifts. In addition, a small and decreasing drop in the C field correction, corresponding to several parts in  $10^{15}$  in the cesium frequency, was observed following each C field reversal. At the conclusion of the measurements the C field correction, calculated from

the mean of six low frequency, (4, -4) ↔ (4, -3) resonances, showed no change within two parts in 10<sup>15</sup> on reversion to the normal C field direction.

Although measurement of the cavity phase difference frequency shift and the second order Doppler shift are not necessary in the determination of the Millman effect frequency shift, since the latter can be measured solely by C field reversal, beam reversal can cause alterations in both the other shifts. It is therefore useful in discussing the various inter-relationships between these quantities to describe them algebraically as follows. If the fractional frequency values of the shifts arising from the Millman effect, the cavity phase difference, the C field, and the second order Doppler effect are given by  $\delta f_m$ ,  $\delta f_c$ ,  $\delta f_h$ , and  $\delta f_d$ , the beam directions by subscripts AB and BA, the C field directions by N and R, referring to normal and reversed, then the difference between the normalized frequency of CsV,  $f_{CsV}$  and that of the maser reference,  $f_M$ , is given as follows, with the various K values accounting for maser drift and the numerical constants associated with the electronic frequency multiplication and synthesis techniques employed:

$$\delta f_m + \delta f_c + \delta f_h \text{ AB,N} + \delta f_d = f_{CsV \text{ AB,N}} - f_M + K_1 \quad (10a)$$

$$-\delta f_m - \delta f_c + \delta f_h \text{ BA,N} + \delta f_d = f_{CsV \text{ BA,N}} - f_M + K_2 \quad (10b)$$

$$-\delta f_m + \delta f_c + \delta f_h \text{ AB,R} + \delta f_d = f_{CsV \text{ AB,R}} - f_M + K_3 \quad (10c)$$

$$\delta f_m - \delta f_c + \delta f_h \text{ BA,R} + \delta f_d = f_{CsV \text{ BA,R}} + f_M + K_4 \quad (10d)$$

In these equations,  $\delta f_c$  and  $\delta f_m$  are arbitrarily chosen as those referring to the AB beam direction and normal, N, C field direction. In addition, since the Ramsey resonance patterns for the two beam directions are so nearly identical, both  $\delta f_c$  and  $\delta f_m$  are assumed only to change sign with beam reversal, and  $\delta f_d$  is assumed to be the same for both beam directions.

Reference to equations 10a - 10d shows that it is indeed possible to determine  $\delta f_m$  simply by C field reversal alone, the beam direction being maintained constant. In the April, 1975, re-evaluation of systematic errors, measurements were made corresponding to the equation pairs 10a and 10c, and 10b and 10d, for two different power levels, one corresponding to that for maximum transition probability,  $P_0$ , and the other at  $P_0 - 3$  dB. The results are shown in Table 1.

Table I. The Millman effect frequency shift expressed in parts in  $10^{13}$  for two different exciting power levels and both beam directions.

Power Level	Beam Direction	
	AB	BA
$P_0$	2.1	1.9
$P_0 - 3$ dB	1.9	2.1

Within the precision of measurement, which is of the order of  $1 \times 10^{-14}$ , no significant power dependence nor dependence on beam direction is apparent, and the mean value is  $2.0 \times 10^{-13}$ . For the September re-evaluation, carried out only at  $P_0 - 3$  dB, the mean frequency shift was also  $2.0 \times 10^{-13}$ .

#### IV. DISCUSSION

The observed Millman effect frequency shift must now be explained in terms of imperfections in the mechanical construction of CSV.

The magnitude of this effect indicates that the required relative angular rotation between orienting and oscillating fields is extremely small, of the order of a few tens of microradians. Such a rotation could conceivably be produced in the vicinity of the waveguide interaction regions by a number of factors, all dependent on the C field generated by the four current-carrying rods, and hence all dependent on the direction of current flow through these rods. Such factors could constitute perturbations arising from the open ends of the cylindrical magnetic shields, those from the current carrying leads feeding the four-rod structure, those due to the slots cut through the shields to permit entry of the waveguide cavity, and also variations in permeability of the shields along the longitudinal seams or at the spot welds. All these factors would in general give rise to a three dimensional curvature of the C field which would be similar at both the waveguide interaction regions. It is known also that the C field, averaged over 30-cm lengths between the two excitation regions and measured by means of the  $(4, -4) \leftrightarrow (4, -3)$  resonances, exhibits variations of about  $\pm 0.06\%$ .

However, despite the existence of these unknown but quite possibly significant sources of C field curvature, it is of interest to estimate the degree of curvature to be expected in the magnetic field

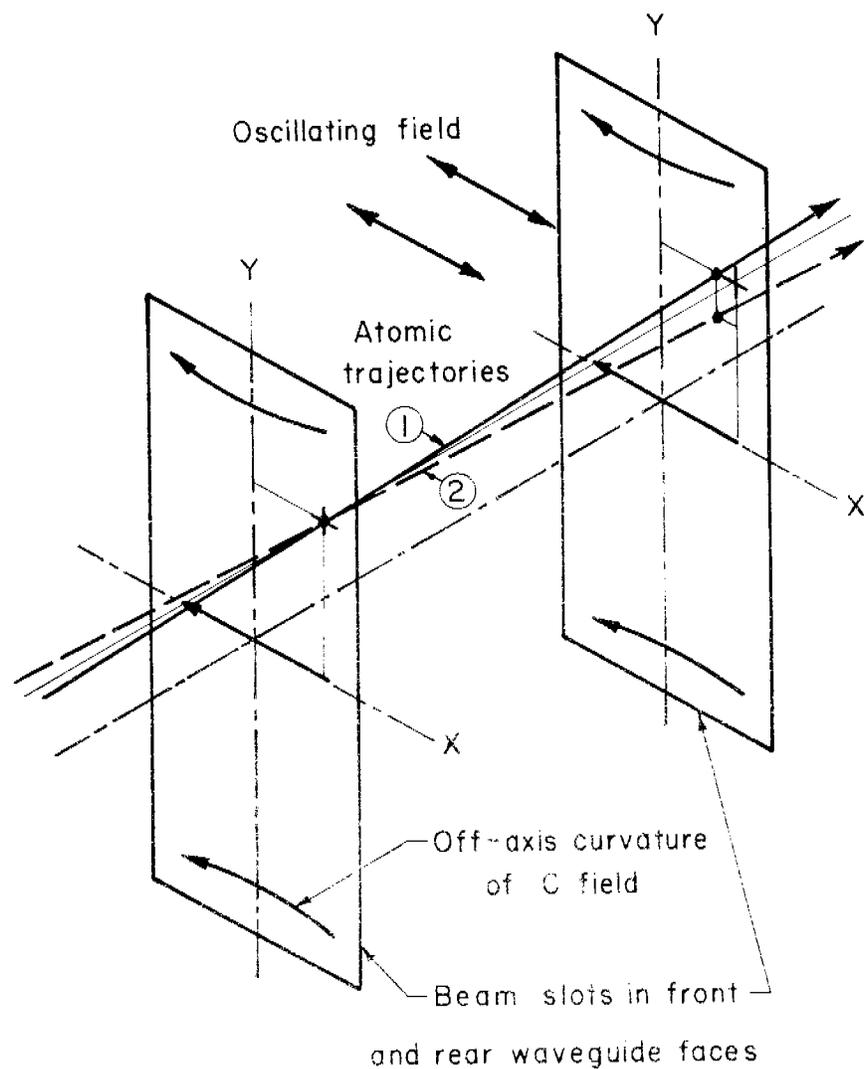


Figure 3. Atomic trajectories through the microwave interaction region for which the angle between orienting and oscillating fields undergoes a continuous change.

produced by the four-rod structure used in CsV. It should be borne in mind, though, that this could be outweighed by that resulting from the uncalculable sources mentioned above, and consequently, little more than an order of magnitude calculation is truly significant. With this proviso, this source of curvature will now be examined in detail. The requirement is that a gradual change in the angle between oscillating and orienting fields occur as the atom progresses through each inter-

action region.

Figure 3 shows two possible atomic trajectories which could result in such a change. One case characterizes trajectories for which the atom maintains its y coordinate throughout its passage from oven to detector, and only the x coordinate changes, e.g., an atom which leaves the upper section of the oven collimator, passes through the upper section of the central slit, and is collected on the upper section of the hot wire detector. The other case characterizes trajectories for which both the x and y coordinates change, e.g., an atom which leaves the upper section of the oven collimator and drops linearly in its passage through the central slit and is collected on a lower portion of the hot wire detector. In general, both types of trajectories occur, and in addition, all trajectories will be more or less parabolic, depending on the atomic velocities, as a result of gravitational acceleration. The general case is, therefore, quite complicated, and consequently only a semi-quantitative analysis will be attempted.

In CsV, the C field is produced by a four-rod current-carrying structure symmetrically located within three concentric molybdenum permalloy magnetic shields of diameters 20.3, 25.4, and 30.5 cm. The x and y spacings of these rods are 8.045 and 13.969 cm respectively, with the x coordinate indicating the C field direction. These spacings were chosen by computer analysis of the near-axis magnetic field with the criterion of minimal deviation of the total magnetic field at any point within the normal beam deflection cross section from that existing on axis. The calculations were initially performed for free-space location of the four current carrying rods, and a later check with allowance made for the presence of the innermost magnetic shield, but not for the effect either of finite permeability of this shield nor for the effect of the waveguide slots through it, indicated that the magnetic field uniformity was improved by it.

Calculations were also made of the angle between the total magnetic field and the x-axis, for free-space location of the four rods, and these results are shown in Table II. Examination of this table shows that this angle varies from zero on the x and y axes to a maximum of about 9 microradians over a cross section limited by  $0 < |x| < 2$  mm,  $0 < |y| < 5$  mm which corresponds approximately to the area of the slot cut into the waveguide cavity to allow passage of the cesium beam. Because of the wide variation of this angle, as indicated in the table, and the actual atomic beam, any explanation based on this table can only be qualitative.

Since each cavity is about 1 cm wide in the beam direction, and is located about 106 cm from the central slit, atoms traversing the cavity must undergo a change in their x coordinate of about 1% during their passage from one waveguide face to the other. Because of the

Table II. Angle between total C field and x-axis, microradians

y-mm	x-mm										
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.00	0.11	0.22	0.34	0.47	0.60	0.74	0.90	1.08	1.27	1.48
1.0	0.00	0.21	0.42	0.64	0.88	1.13	1.40	1.71	2.04	2.41	2.82
1.5	0.00	0.28	0.56	0.86	1.17	1.52	1.89	2.31	2.78	3.30	3.88
2.0	0.00	0.30	0.61	0.94	1.30	1.69	2.13	2.62	3.18	3.81	4.52
2.5	0.00	0.27	0.55	0.86	1.20	1.58	2.02	2.53	3.12	3.80	4.59
3.0	0.00	0.17	0.35	0.56	0.81	1.12	1.49	1.95	2.50	3.16	3.95
3.5	0.00	-0.02	-0.02	0.01	0.08	0.23	0.45	0.76	1.19	1.75	2.45
4.0	0.00	-0.31	-0.60	-0.85	-1.04	-1.17	-1.20	-1.12	-0.91	-0.56	-0.04
4.5	0.00	-0.71	-1.40	-2.04	-2.63	-3.13	-3.53	-3.81	-3.93	-3.90	-3.68
5.0	0.00	-1.24	-2.46	-3.63	-4.73	-5.74	-6.64	-7.40	-7.99	-8.40	-8.60

presence of 1 mm diameter stop wires at the innermost ends of both state selector magnets, the inverse variation with velocity of the deflection of the atoms from the axis, and the ribbon cross-section of the beam, most of the atoms will be in the region defined by  $1.0 \text{ mm} < |x| < 2.0 \text{ mm}$  and  $|y| < 3.5 \text{ mm}$ . Except for atoms near the x axis, it is apparent that changes of only a few hundredths of a microradian occur for atoms following most of the possible trajectories, including those for which the y coordinate changes as well. For example, an atom which enters the excitation region at  $x = 1.600 \text{ mm}$ ,  $y = 2.000 \text{ mm}$ , and leaves at  $x = 1.584 \text{ mm}$ ,  $y = 2.000 \text{ mm}$ , experiences a change in angle between oscillating and orienting fields of about 0.05 microradian. For atoms closer to the y axis, the change is smaller, and for those farther away from the y axis, it is greater. However, it is clear that in the case of the four rods located in free space, the angular changes to be expected for typical atomic trajectories are about 3 orders of magnitude too small to explain the observed Millman frequency shift.

As mentioned earlier, however, the presence of the magnetic shields, and particularly the field leakage through both the waveguide slots and the open ends of the shields, can be expected to alter to some degree the magnetic field produced by the four rods. In addition, the experimental observation that the C field varies by  $\pm 0.06\%$  from one 30 cm length to another between the interaction regions indicates most probably that there are significant variations in the effective permeability of the innermost shield. Apparently, all these perturbations combine to produce inhomogeneities and curvature of the C field at the cavity beam slots such as to give rise to the Millman effect measured.

Table III. Millman effect frequency shift, mHz, for lower and upper velocity limits of  $1.0\alpha$  and  $1.8\alpha$ , characteristic of normal operation of CsV.

Exciting Power, $\alpha/\ell$	Frequency Offset, Hz									
	5	10	15	20	25	30	35	40	45	50
0.2	1.92	1.88	1.91	1.92	1.96	1.92	1.96	1.98	2.02	2.08
0.4	1.75	1.77	1.77	1.77	1.79	1.79	1.82	1.85	1.90	1.99
0.6	1.78	1.82	1.75	1.82	1.80	1.82	1.83	1.87	1.91	2.01
0.8	1.75	1.81	1.78	1.82	1.80	1.82	1.86	1.89	1.94	2.03
1.0	1.81	1.94	1.84	1.87	1.88	1.89	1.90	1.93	1.98	2.05
1.2	1.92	1.88	1.91	1.92	1.96	1.92	1.96	1.98	2.02	2.08

Despite this somewhat unsatisfactory explanation of the origin of this effect, it is of interest to calculate from the theory presented earlier a mean value of  $\gamma$  required to account for the measured Millman effect frequency shift of  $2.0 \times 10^{-13}$ , or about 0.0018 Hz in the cesium transition frequency. The physical significance of such a mean value of  $\gamma$  is, however, open to question. Table III shows results of calculations based on this theory for a value of  $\gamma$  of 45.3 micro-radians, which was chosen so as to provide agreement with experimental observations for the normal operating point of CsV, characterized by a servo frequency offset of about 35 Hz and a value of  $\alpha/\ell$  of 0.8. This table shows that only a small power dependence should be expected, the Millman effect frequency shift changing by less than 0.0001 Hz between values of  $\alpha/\ell$  of 0.8 and 1.0, which correspond to power levels of  $P_0 - 3$  dB and  $P_0$ . A similar power dependence was in fact measured, indicating that the concept of a mean value of  $\gamma$  may actually have physical significance.

This apparent lack of dependence on either exciting power level or offset frequency for CsV in its normal mode of operation should not, however, be construed as indicating that in a more general case with a wider velocity distribution no such dependence exists. Table IV shows values of the Millman effect frequency shift calculated for the same physical constants characteristic of CsV, but with a wider velocity distribution having upper and lower velocity limits of  $2.4$  and  $0.4 \alpha$ . Such a distribution could be obtained in CsV by use of state selector magnets having smaller magnetic field gradients and by eliminating the stop wires. The entries in this table vary by a factor greater than 2. Consequently, the use of wide velocity distributions and somewhat less uniform C fields could, in standards such as CsV, lead to very significant power and frequency offset dependence of the Millman effect frequency shift. These two velocity distributions are shown in figure 4.

Table IV. Millman effect frequency shift, mHz, for lower and upper velocity limits of  $0.4\alpha$  and  $2.4\alpha$ , which could result from operation of CsV without stop wires and with weaker state selector magnets.

Exciting Power, $\alpha/\lambda$	Frequency Offset, Hz						
	5	10	15	20	25	30	35
0.2	1.04	1.09	1.15	1.27	1.50	2.03	3.89
0.4	1.11	1.18	1.24	1.34	1.50	1.79	2.34
0.6	1.41	1.37	1.38	1.46	1.56	1.66	1.90
0.8	1.52	1.56	1.54	1.66	1.66	1.75	1.85
1.0	1.57	1.61	1.65	1.76	1.87	1.96	2.04
1.2	1.64	1.66	1.74	1.82	1.99	2.06	2.22

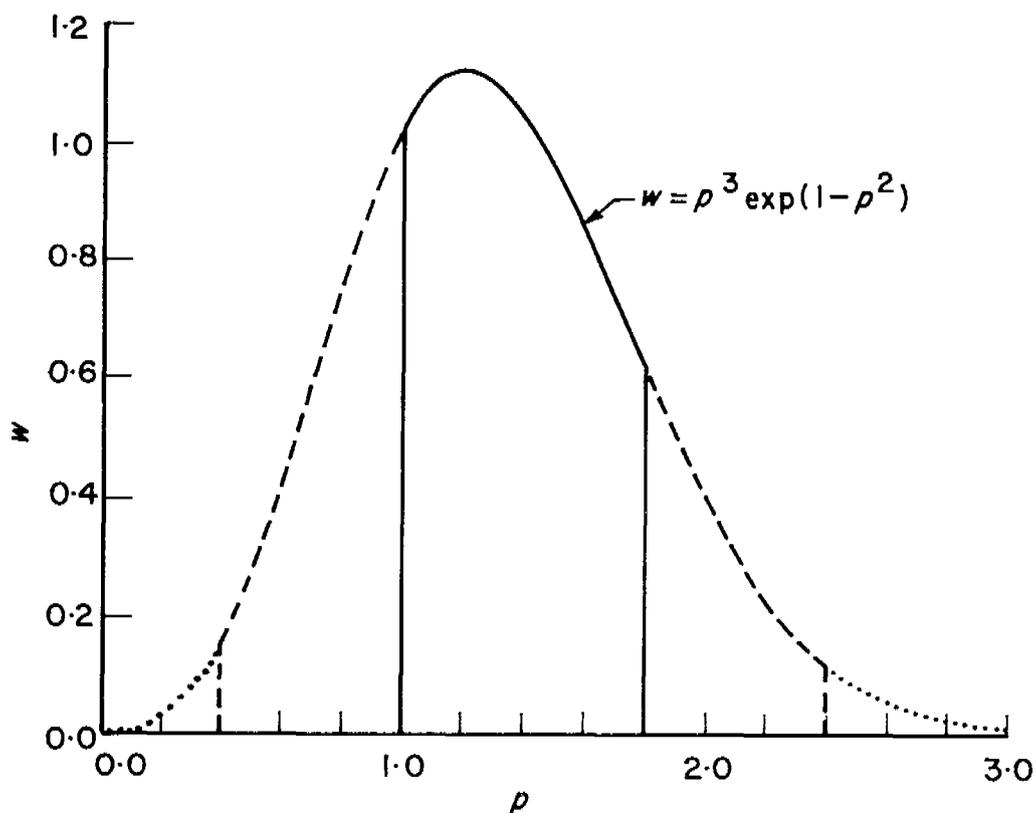


Figure 4. Truncated Maxwellian velocity distributions of atoms in the beam,  $w = p^3 \exp(1 - p^2)$ , for normal operation of CsV with  $p_{\min} = 1.0$  and  $p_{\max} = 1.8$ , and for wider velocity limits with  $p_{\min} = 0.4$  and  $p_{\max} = 2.4$ .

For commercial cesium standards with wide velocity distributions, greater resonance line widths, and probably greater C field non-uniformities, it is likely that the Millman effect could cause significantly large systematic errors. These could also be time-dependent as the result of changes in the magnetization of the magnetic shields.

It was pointed out earlier that reversal of the C field constituted the only means of separating the Millman and cavity phase difference frequency shifts. Reference to equations 10a - d shows that it is possible for these two effects to partially cancel each other, depending on the C field and beam directions employed. If only the beam direction is reversed in the evaluation of systematic errors, with the consequence that  $\delta f_m$  and  $\delta f_c$  are not resolved independently, then it is evident that quite an erroneous estimate of the cavity phase difference frequency shift and its power dependence could be made. Errors in such an estimate would be expected to increase with increasing resonance line width, and hence to be of particular importance in standards, with short interaction lengths. Even for long beam standards, a cavity phase difference frequency shift approaching  $10^{-12}$  is not unreasonable. For example, if the values of  $\delta f_c$  and  $\delta f_m$  were actually  $1.0 \times 10^{-12}$  and  $0.9 \times 10^{-12}$ , then for a particular beam and C field direction, the combined effect would be only  $1 \times 10^{-13}$ , which would lead to an order of magnitude error in the measurement of the cavity phase difference, with concomitant errors in the power dependence. It is thus evident that in the evaluation of systematic errors in cesium beam standards it is essential that measurements be made for both directions of the C field as well as for both directions of the beam.

## V. CONCLUSIONS

The Millman effect as a limiting systematic error in cesium beam atomic frequency standards, and specifically in CsV, has been discussed and means of measuring it outlined. A possible explanation for it has been proposed, and calculations based on this explanation are in reasonable agreement with experiment. It is apparent that an understanding of the Millman effect is important if accuracies of the order of  $10^{-13}$  are to be attained in long-beam primary laboratory cesium standards. It is probably even more important in the case of short-beam commercial standards which would be expected to be much more susceptible to frequency shifts arising from this effect. It is in fact possible that the frequency jumps and drifts which have been observed in these standards may be related to the Millman effect.

## ACKNOWLEDGEMENTS

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#### REFERENCES

1. Hellwig, H.W.: Proc. IEEE 63, 212 (1975).
2. Mungall, A.G., Bailey, R., Daams, H., Morris, D., Costain, C.C.: Metrologia 9, 113 (1973).
3. Ramsey, N.F.: Molecular Beams, 128, 139, 155, Oxford University Press (1956).
4. Millman, S.: Phys. Rev. 55, 628 (1939).
5. Mungall, A.G.: Metrologia 7, 49 (1971).
6. Mungall, A.G.: Metrologia 8, 28 (1972).
7. Hahn, S.L.: Bulletin de l'Academie Polonaise des Sciences 23, 249 (1975).

QUESTION AND ANSWER PERIOD

DR. HELLWIG:

A comment on the Millman effect. First, I do not doubt that the measurement, of course, is as precise as Dr. Costain reported. We have sort of looked into the Millman effect at NBS and came to the conclusion without going into much theoretical detail now, that it cannot be the Millman effect which explains your measurements because the Millman effect--

DR. COSTAIN:

Would you repeat that?

DR. HELLWIG:

That the Millman effect is not the right vehicle to explain your measurements because the Millman effect should not be of any significant magnitude for magnetic field independent transitions  $\Delta MF = 0$  transitions. The basic physical reason for that is that the orientation of the dipole moment of the flying atoms is random with regard to being perpendicular to the orienting magnetic field.

DR. COSTAIN:

There is some disagreement on this factor, I know, and I think we continue to disagree. I think it is the Millman effect we are seeing. One of our difficulties is that our correspondence has been rather limited in the past eight weeks. We have not yet had an opinion from Dr. Hahn.

DR. HELLWIG:

I know that Dr. Hahn's paper is the theoretical foundation to that, so I have to add to that that we disagree with Dr. Hahn's paper.

MR. ALLAN:

One comment that may affect many users of commercial cesium that I would like to make. We have found and, of course, HP now has in the high performance tube, the ability to degauss the unit. When you do change the C field in a standard there is a residual which takes time to accommodate domain changes in the surrounding shielding and this some-

times can be a problem when measuring the reversal of the C field, it seems that would be an entry, at least in long term stability.

I don't know whether you have encountered that or not.

DR. COSTAIN:

Yes, it is.