FREQUENCY STABILITY REQUIREMENTS FOR TWO WAY RANGE RATE TRACKING

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ABSTRACT

Accuracy limitations to two way range rate doppler tracking due to master (reference) oscillator frequency instabilities are discussed. Theory is developed to treat both the effects of random and non-random oscillator instabilities. The non-random instabilities treated are drift, environmental effects, and coherent phase modulation. The effects of random instabilities on range rate accuracy are shown to be describable in terms of $\sigma_y(2, T, \tau)$. For the typical noise processes encountered in precision oscillators, range rate error is related to the more familiar $\sigma_y(\tau)$ and $\mathcal{L}(f)$. Three examples are discussed to show how to determine range rate error from given $\sigma_y(\tau)$ or $\mathcal{L}(f)$ curves, and approximations are developed to simplify the treatment of complex systems. An error analysis of range determined from range rate data is also given.

INTRODUCTION

Two way range rate doppler tracking (TWDT) is a means of measuring range rate by observing the doppler shift in a radio signal coherently transponded from a satellite. Recently proposed applications of TWDT to Earth and ocean physics have range rate accuracy requirements of 0.003 to 0.005 cm/sec.[1, 2] These stringent accuracy requirements impose constraints on TWDT systems which make careful analysis and minimization of system errors imperative. One source of error in TWDT systems is the instability of the master oscillator (reference oscillator). Others have analyzed the effects of this error source,[3,4,5] but have either not treated the problem in sufficient detail, made some errors in their analysis, or not treated the problem directly as it applies to TWDT systems for satellites. This paper will attempt to correct these defects. The paper will not discuss individual TWDT systems, but will take a generalized approach. Existing systems and requirements, however, will be kept in mind to ensure the utility of the paper's results.

STATISTICS OF RANGE RATE ERROR

A generalized schematic of a TWDT system is shown in Figure 1. In a TWDT system, a reference frequency, f_0, provided by the master oscillator is transmitted to the satellite and transponded back to the receiver. The doppler shifted frequency, $f_0 + f_D$, is then mixed with f_0 to produce f_D. After passing through a band pass filter, f_D is measured in a frequency or period counter. In actual systems, f_D is biased by an arbitrary frequency[4] so that the doppler signal can be averaged over any time, T. The master oscillator also supplies the time base for the counter. The accuracy requirements for this are not stringent, and will not be treated here.

The counter used to measure f_D can fit into four possible categories.[4] The first two categories deal with the way individual measurements are taken. The

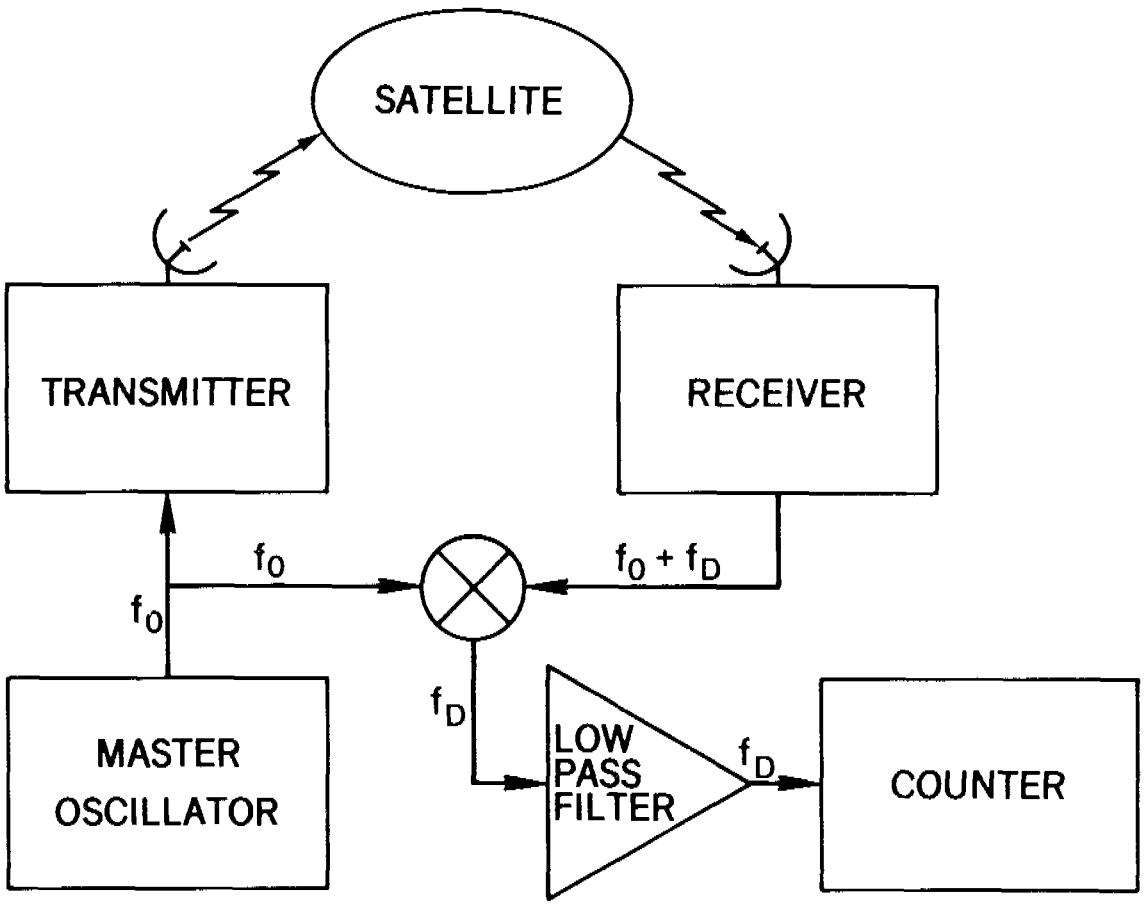

Fig. 1—Two way doppler tracking

counter can measure the total number of periods of f_D for a fixed elapsed time, T, or it can measure the total elapsed time for a fixed number of periods. In this paper, we shall consider T given; the results can be related to the fixed period case through an estimate of the behavior of f_D. The other two categories deal with the way in which successive measurements are treated. The counter is considered destructive or non-destructive depending on whether it is cleared or not cleared after each measurement. In the destructive case, the counter acts as a conventional period or frequency counter. In the non-destructive case, the counter acts as a real time clock. The effects of destructive versus non-destructive data taking will be discussed later.

Nominally, when f_0 is subtracted from $f_0 + f_D$, all that remains is f_D. The tracking signal, however, takes a finite time, τ, to propagate from the transmitter to the receiver. If the frequency of the master oscillator shifts, in time τ, by δf_0, the counter will measure $f_D + \delta f_0$ as the doppler shift. Since there is no way to distinguish between f_D and δf_0 in the counter, δf_0 will introduce a range rate error.

No matter how frequencies are coherently changed as signals propagate through the TWDT system, the doppler shift will be given by[6]:

$$\frac{f_D}{f_0} = \frac{2v}{c} \tag{1}$$

where v is the satellite's range rate and where c is the velocity of light. Using (1), it can be shown that the range rate error from master oscillator instability for averaging time, T, is given by:

$$\delta v(t, T, \tau) = \frac{c}{2T} \left[\int_{t+\tau}^{t+T+\tau} y(t')\, dt' - \int_{t}^{t+T} y(t')\, dt' \right] \tag{2}$$

y (t) is the normalized master oscillator frequency change:

$$y(t) = \frac{f_0(t) - f_0}{f_0}$$

and f_0 is the nominal oscillator frequency. For $\tau < T$, this can be rewritten as (see Figure 2):

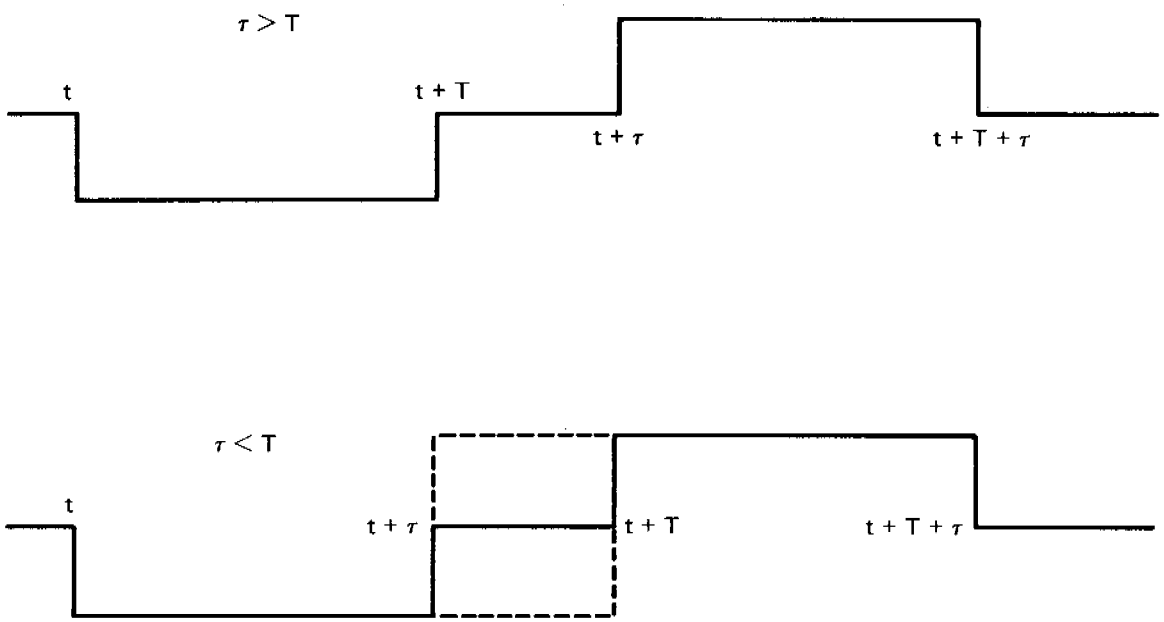

Fig. 2—Response functions for range rate error

$(\tau < T)$

$$\delta v(t, T, \tau) = \frac{c}{2T}\left[\int_{t+T}^{t+\tau+T} y(t')\,dt' - \int_{t}^{t+\tau} y(t')\,dt'\right] \tag{3}$$

If the master oscillator instabilities are random, range rate errors can be described in terms of the variance of $\delta v(t, T, \tau)$:

$$\sigma_v^2(T, \tau) = \left\langle \left(\delta v(t, T, \tau)\right)^2 \right\rangle.$$

$\langle A \rangle$ denotes the average of A over time, t. Using equation 3, for $\tau < T$, the variance becomes:

$(\tau < T)$

$$\sigma_v^2 = \frac{c^2}{2}\left(\frac{\tau}{T}\right)^2 \sigma_y^2(2, T, \tau) \tag{4}$$

where $\sigma_y^2(2, t_1, t_2)$ is the two sample Allan variance of $y(t)$ for averaging time t_2 and dead time $t_1 - t_2$ given by[7,8]:

$$\sigma_y^2(2, t_1, t_2) = \frac{1}{2} \left\langle \left(\int_t^{t+t_2} y(t')\, dt' - \int_{t+t_1}^{t+t_1+t_2} y(t')\, dt' \right)^2 \right\rangle$$

For $\tau \geq T$, the variance is:

$$(\tau \geq T)$$

$$\sigma_v^2 = \frac{c^2}{2} \sigma_y^2(2, \tau, T) \tag{5}$$

Notice that in the case of (5), the definitions of τ and T are reversed from those of references 7 and 8.

In general, $\sigma_y^2(2, T, \tau)$ is not a statistic which is given in oscillator specifications. In the time domain, generally, $\sigma_y^2(\tau) = \sigma_y^2(2, \tau, \tau)$ is what is specified. One can relate these two statistics by[7,8]:

$$\sigma_y^2(2, T, \tau) = B_2(r, \mu)\, \sigma_y^2(\tau) \tag{6}$$

where $r = T/\tau$, and where μ characterizes the noise process involved.

Another typical way in which an oscillator is specified is by $\mathcal{L}(f)$, the power density in one phase modulation side band divided by the total oscillator power.[8] $\mathcal{L}(f)$ can be related to $\sigma_y^2(2, T, \tau)$ by[8]:

$$\sigma_y^2(2, T, \tau) = \frac{4}{(\pi f_0 \tau)^2} \int_0^\infty df\, \mathcal{L}(f)\, |h_\ell(f)|^2\, H(f) \tag{7}$$

where $h_\ell(f - f_D)$ is the frequency response function of the post mixer filter of Figure 1, and where:

$$H(f) = \sin^2(\pi f \tau) \sin^2(\pi f T)$$

TYPICAL OSCILLATOR INSTABILITIES

Master oscillator frequency instabilities can be divided into two classes, random and non-random effects. We shall first consider two non-random effects, frequency drift and coherent phase modulation.

An oscillator's frequency drift is defined by:

$$y(t) = Dt$$

Using this and equations 2 and 3, one can show that for both $\tau < T$ and $\tau \geq T$:

$$\delta v = \frac{c\,D}{2}\,\tau \tag{8}$$

If we use $\delta v = 0.001$ cm/sec as a target error and $\tau = 0.3$ sec as a typical worst case delay, we obtain for D a maximum allowable value of:

$$D = 1.92 \times 10^{-8}\ /\text{day}$$

Clearly this is easily met with most good crystal oscillators.

Equation 8 can also be used to determine environmental constraints if the frequency change per unit environmental change is known. If, for example, an oscillator changes its fractional frequency by 1×10^{-12} per °C, and $\delta v \leq 0.001$ cm/sec is required, one can write:

$$D = 1 \times 10^{-12}/\,°C \times K\ (\,°C/\,\text{sec})$$

Using equation 8, we obtain:

$$K \leq 0.22\ °C/\,\text{sec}$$

How coherent phase modulation of the master oscillator effects a TWDT system depends not only on the modulation amplitude and frequency, but also on the precise phase relationships between the modulation and the data taking process.

We can, however, obtain approximate results by ignoring phase, and describing the modulation by:

$$\mathcal{S}_M(f) = \frac{1}{2} P_M \, \delta(f - f_M)$$

where f_M is the modulation frequency, and where $1/2 \; P_M$ is the fractional power in one modulation sideband. Using this in (7), we obtain:

$$\sigma_y^2(2, T, \tau) = \frac{2 P_M}{(\pi f_0 \tau)^2} \, |h_\ell(f_M)|^2 \, H(f_M)$$

Since $H(f) \leq 1$, we obtain the inequality:

$$\sigma_y^2(2, T, \tau) \leq \frac{2 P_M}{(\pi f_0 \tau)^2} \, |h_\ell(f_M)|^2$$

Substituting this in (4) and (5), gives:

$(\tau < T)$

$$\sigma_v^2 \leq \frac{c^2}{(\pi f_0 T)^2} \, |h_\ell(f_M)|^2 \, P_M \tag{9}$$

and:

$(\tau \geq T)$

$$\sigma_v^2 \leq \frac{c^2}{(\pi f_0 \tau)^2} \, |h_\ell(f_M)|^2 \, P_M \tag{10}$$

As an example, let $\sigma_v = 0.001$ cm/sec, $\tau = 0.3$ sec, $T = 5$ sec, and $f_0 =$ 5 M Hz. We obtain:

$$|h_\ell(f_M)|^2 \, P_M \simeq -112 \text{ db}$$

as the estimated upper limit of filtered phase modulation that can be tolerated.

Random processes typically encountered in precision oscillators can be described as weighted and filtered sums of power spectral densities of y (t) given by[7, 8]:

$$S_y(f) = \frac{h_\alpha}{f^\alpha}$$

with:

$$\alpha \qquad 2, 1, 0, -1, -2$$

S_y (f) can be related to $\mathcal{L}$(f) by[8]:

$$\mathcal{L}(f) = \frac{1}{2}\left(\frac{f_0}{f}\right)^2 S_y(f) \tag{10}$$

Using (10), (4), (5), and previously derived equations relating $\sigma_y^2(\tau)$ and $\sigma_y^2(2, T, \tau)$ for each noise process,[9] one can relate $\mathcal{L}$ (f), $\sigma_y(\tau)$ and $\sigma_y^2(T, \tau)$. Chart 1 gives these relationships. The chart assumes that the post-mixer band pass filter is infinitely sharp with a pass band of $f_D - f_h$ to $f_D + f_h$, that $2\pi f_h \tau >> 1$, and that $2\pi f_h T >> 1$. For completeness, drift is included even though it is not a random process. To simplify formulae in the chart, the following are used:

$$r = \frac{T}{\tau}$$

$$x = \int y\, dt = \frac{\text{phase}}{2\pi f_0}$$

$$\delta_{r,1} = \begin{cases} 1 \text{ for } r = 1 \\ 0 \text{ for } r \neq 0 \end{cases}$$

Chart 1—Range Rate Error for Various Noise Processes

Process	$\mathcal{L}(f)$	$\sigma_y^2(\tau)$	$r = \frac{T}{\tau}$ $\delta_{r,1} = \begin{cases} 1 & r = 1 \\ 0 & r \neq 1 \end{cases}$ $\sigma_v^2(T, \tau)$
White x $S_y = h_2 f^2$	$\frac{h_2 f_0^2}{2}$	$\frac{3 h_2 f_h}{(2\pi\tau)^2}$	$\frac{h_2 f_h c^2 \left(1 + \frac{1}{2}\delta_{r,1}\right)}{(2\pi T)^2}$
Flicker x $S_y = h_1 f$	$\frac{h_1 f_0^2}{2f}$	$\frac{h_1}{(2\pi\tau)^2}\left[\frac{9}{2} - \ln 2 + \ln(2\pi f_h \tau)\right]$	$\frac{h_1 c^2}{(2\pi T)^2}\left[\frac{3}{2} + \ln(2\pi f_h \tau)\right]$ $\quad r \gg 1$ $\frac{h_1 c^2}{(2\pi T)^2}\left[\frac{3}{2} + \ln(2\pi f_h T)\right]$ $\quad r \ll 1$
White y $S_y = h_0$	$\frac{h_0 f_0^2}{2f^2}$	$\frac{h_0}{2\tau}$	$\frac{h_0 c^2 \tau}{4T^2}$ $\quad r > 1$ $\frac{h_0 c^2}{4T}$ $\quad r \leq 1$
Flicker y $S_y = \frac{h_{-1}}{f}$	$\frac{h_{-1} f_0^2}{2f^3}$	$h_{-1}\, 2 \ln 2$	$r \geq 1$: $\frac{h_{-1} c^2 \tau^2}{4T^2}\left[(r+1)^2 \ln(r-1) + (r-1)^2 \ln\lvert r-1\rvert - 2r^2 \ln r\right]$ $r < 1$: $\frac{h_{-1} c^2}{4}\left[\left(1+\frac{1}{r}\right)^2 \ln\left(1+\frac{1}{r}\right) - \left(1-\frac{1}{r}\right)^2 \ln\left\lvert 1-\frac{1}{r}\right\rvert + \frac{2}{r^2}\ln r\right]$
Random walk y $S_y = \frac{h_{-2}}{f^2}$	$\frac{h_{-2} f_0^2}{2f^4}$	$\frac{1}{6} h_{-2} (2\pi)^2 \tau$	$\frac{h_{-2}(\pi c)^2 \tau^3}{6T}\left[3r - 1\right]$ $\quad r \geq 1$ $\frac{h_{-2}(\pi c)^2 T}{6}\left[\frac{3}{r} - 1\right]$ $\quad r < 1$
Drift	———	$\frac{D^2 \tau^2}{2}$	$\left(\frac{c D \tau}{2}\right)^2$

Notice that for all processes but random walk y and drift, when $\tau < T$, σ_v^2 is proportional to T^{-2}. By statistically averaging N successively taken counter readings, one improves the variance of the average by N^{-1} or T^{-1}. This means that as long as random walk y or drift processes are not involved, smaller errors are obtained by going to longer averaging times or taking non-destructive data rather than by statistically averaging data. Even when taking non-destructive data, to obtain the smallest σ_v^2 for a given T, one should only use data every T seconds apart, and not least squares fit the data from smaller time intervals.

Notice also that for white x noise, σ_v^2 is proportional to f_h. This allows one to reduce the white noise contribution to σ_v^2 by reducing f_h. Flicker x also depends on f_h, but in such a slowly varying manner that it can essentially be considered fixed.

APPLICATION TO TYPICAL REFERENCE OSCILLATORS

In this section, $\sigma_v(T, \tau)$ shall be derived for three reference oscillators, an Oscilloquartz crystal oscillator, a Hewlett-Packard cesium standard, and a NASA hydrogen maser, to demonstrate techniques for using the theory of the previous sections. The results for the two commercial oscillators used are based on manufacturers specifications. Their use is for the purpose of example only, and does not constitute an endorsement of these products or a confirmation of their specifications.

The first oscillator we will consider is the Oscilloquartz B-5400 crystal oscillator.[10] Its $\mathcal{L}(f)$ spectrum is given in Figure 3 and its $\sigma_y(\tau)$ curve is given in Figure 4. Deriving σ_v for this device is very simple; since its instabilities are just the sum of simple processes (white y, flicker x, flicker y, drift), σ_v^2 is just the sum of the results taken from Chart 1 for each process. Figure 5 shows $\sigma_v(T, \tau)$ for $\tau = 0.3$ s.

The second oscillator to be considered is the Hewlett-Packard 5061A High Performance Cesium Beam Standard.[11] $\mathcal{L}(f)$ for this device is shown in Figure 6. Notice that, in this case, we don't have just the sum of simple processes. Equation 7 could be used to generate σ_v^2 from $\mathcal{L}(f)$, but this tedious method can be avoided; we can quickly obtain an approximate curve for $\sigma_v(T, \tau)$ by using the limiting properties of H(f) in relation to $\mathcal{L}(f)$.

For our approximation, we shall rely on the fact that $\mathcal{L}(f)$ can be broken in two general categories: a short term $\mathcal{L}_s(f)$ which changes slowly with f, and a long term $\mathcal{L}_\ell(f)$ which blows up as f goes to zero and makes a negligible contribution for large f. Using this, we can write:

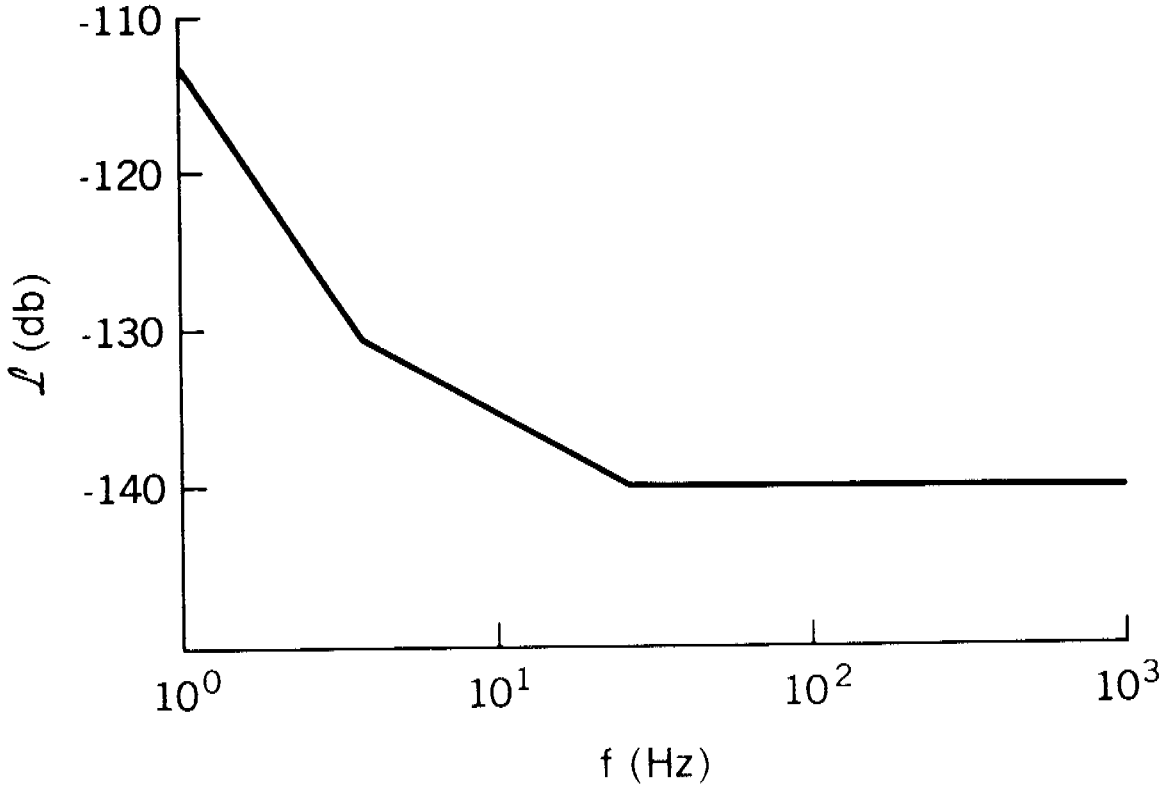

Fig. 3—ℒ (f) for Oscilloquartz B-5400

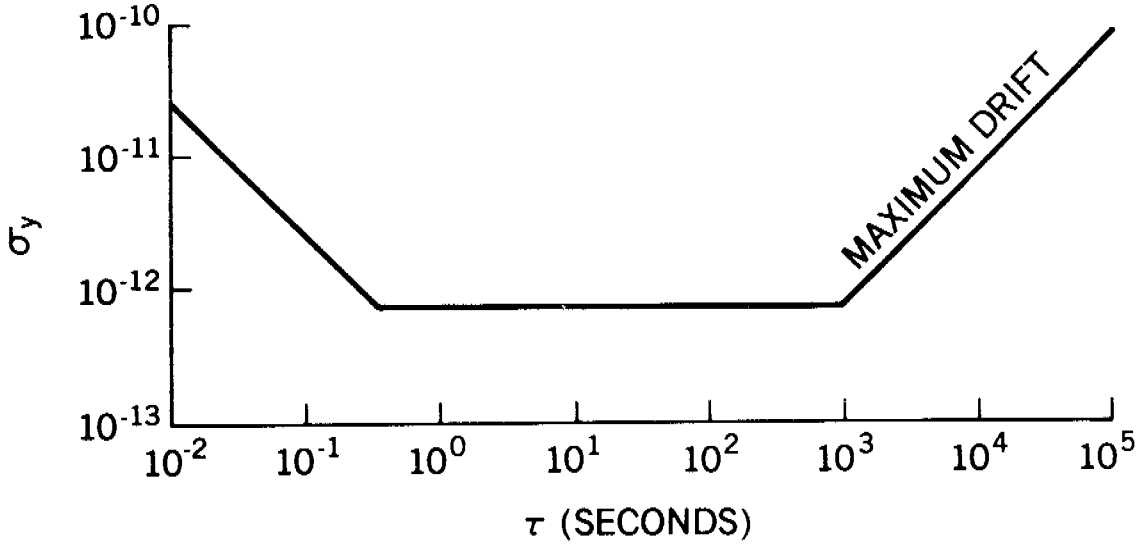

Fig. 4—$\sigma_y(\tau)$ for Oscilloquartz B-5400

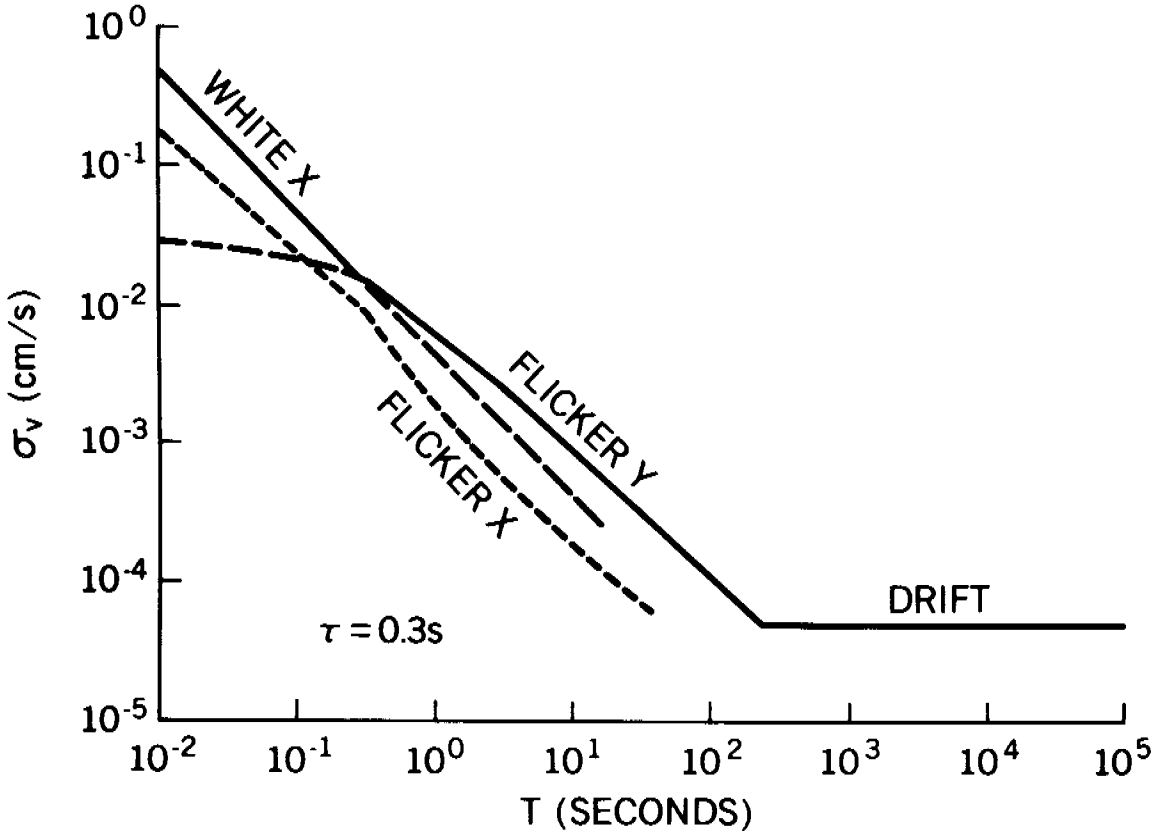

Fig. 5—σ_v (T, 0.3 s) for Oscilloquartz B-5400

$$\sigma_y^2(2, T, \tau) \simeq \sigma_{ys}^2 + \sigma_{y\ell}^2$$

and use (4) and (5) to obtain approximations for σ_v^2.

Since H(f) is a periodic function, for a slowly changing $\mathcal{L}_s$(f), we can replace H(f) $\mathcal{L}_s$(f) in the integral of equation 7 by $< H(f) > \mathcal{L}_s(f)$. $< H(f) >$ denotes the average of H(f). The short term part of equation 7 becomes:

$$\sigma_{ys}^2(2, T, \tau) = \frac{1 + \frac{1}{2}\delta_{r,1}}{(\pi f_0 \tau)^2} \int_0^\infty |h_\ell(f)|^2 \mathcal{L}(f)\, df \qquad (11)$$

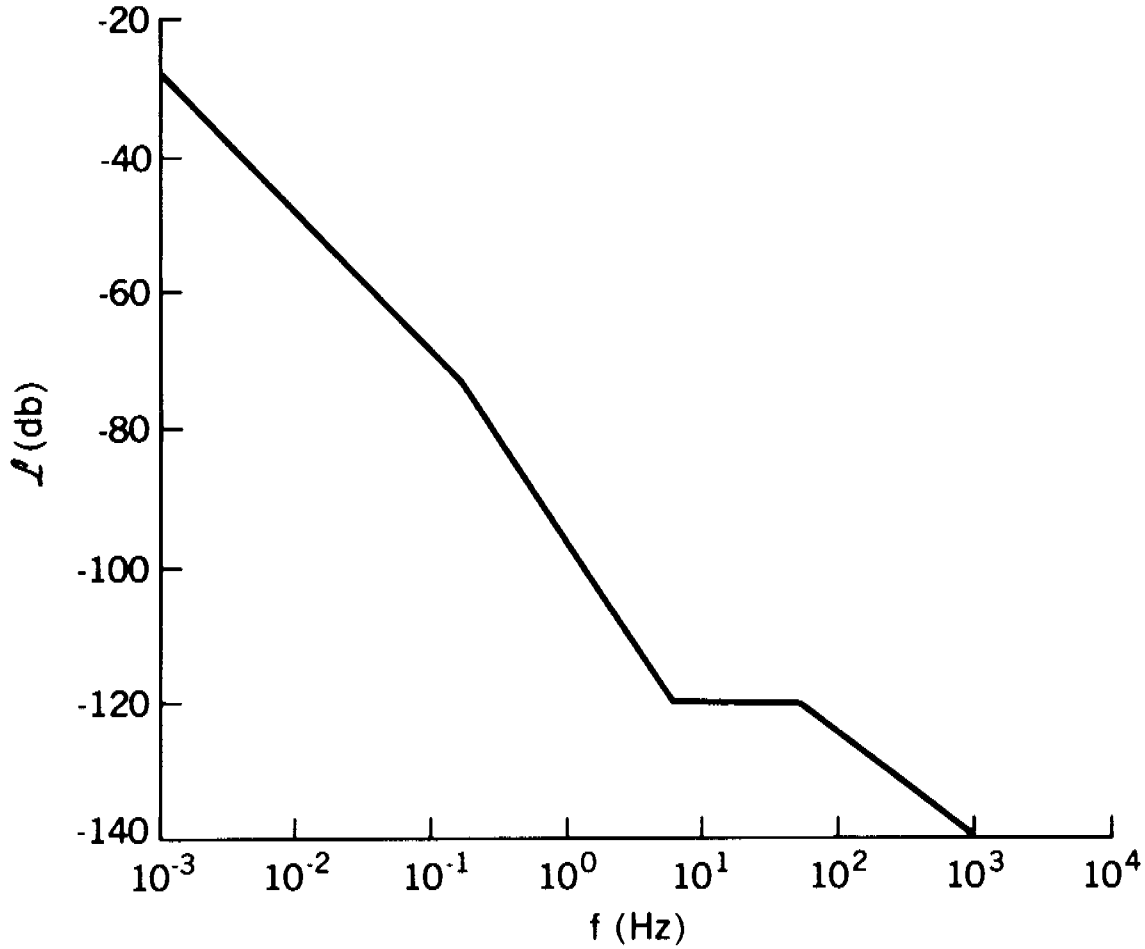

Fig. 6—$\mathcal{L}$(f) for Hewlett-Packard High Performance Cesium Standard

The long term part of $\sigma_y^2(2, T, \tau)$ is determined by H (f) $\mathcal{L}_\ell$ (f) in a region near f = 0. For long term noise processes, $\sigma_y^2(2, T, \tau)$ is determined by H (f) $\mathcal{L}_\ell$ (f) in the regions of f given in the following chart ($\tau < T$):

noise process	region of contribution
white y	$\frac{1}{2T} \lesssim f \lesssim \frac{1}{2\tau}$
flicker y	$f \simeq \frac{1}{2T}$
random walk y	$f \lesssim \frac{1}{2T}$

For a complex $\mathcal{L}_\ell$ (f), which noise process contributes $\sigma_{y\ell}^2$ is determined by the chart and the appropriate values of T and τ. For the Hewlett-Packard cesium standard, $\mathcal{L}$ (f) changes from flicker y to white y noise at f = 0.1 Hz. This means that, for τ = 0.3 s, $\sigma_{y\ell}^2$ is determined by the flicker y part of $\mathcal{L}$(f) for T up to 5 seconds, and by the white y part of $\mathcal{L}$(f) for T greater than 5 seconds. Because white y noise is determined by $\mathcal{L}$ (f) for f up to $1/2\tau$ = 1.67 Hz, and the white y noise is truncated at f = 0.1 Hz, the full value of σ_v^2 from Chart 1 cannot be used to determine the white y contribution for the cesium standard, but must be multiplied by a reduction factor, ρ. An approximate method for calculating ρ is shown in Figure 7. Figure 8 shows σ_v(T, 0.3 s) for the cesium standard using the approximation techniques just derived.

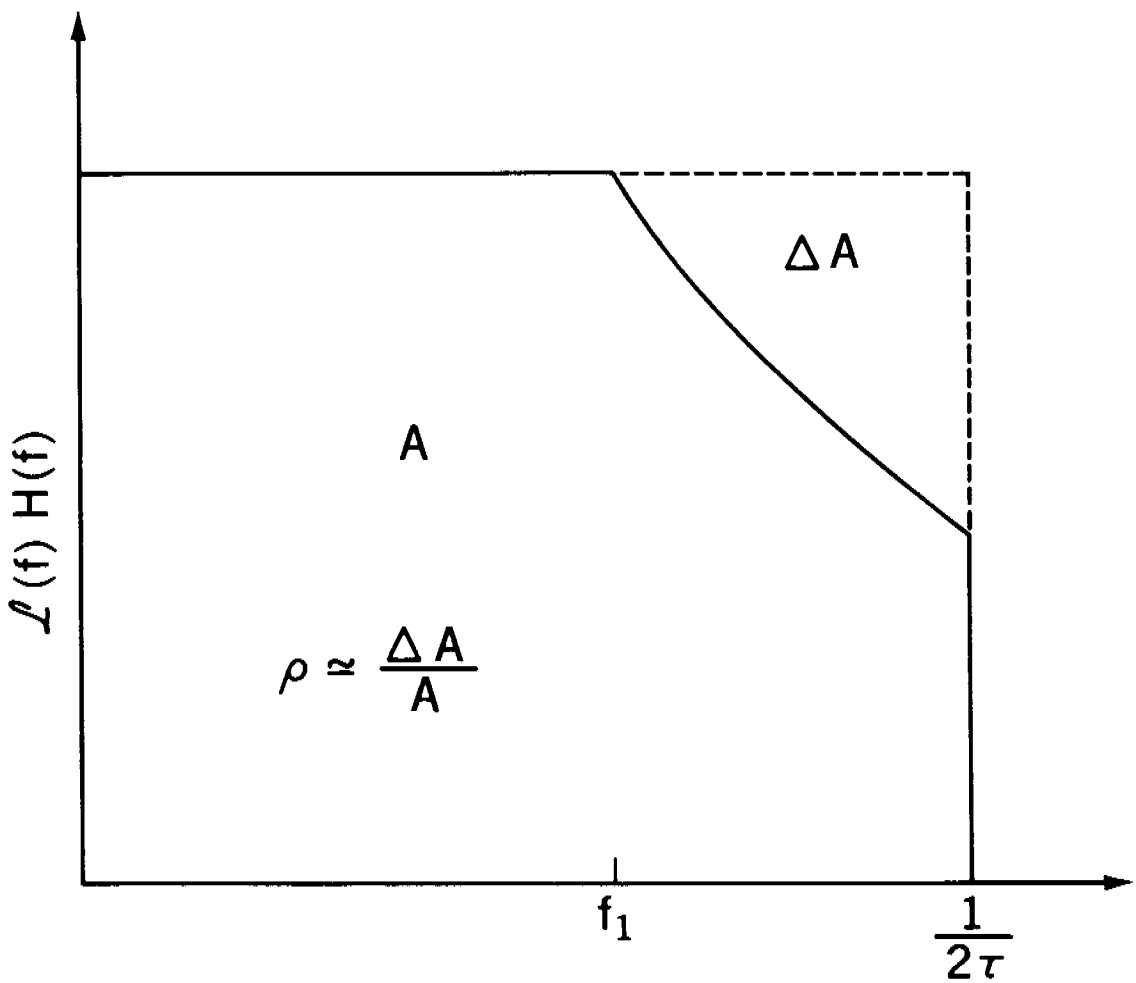

Fig. 7— Approximate calculation of ρ

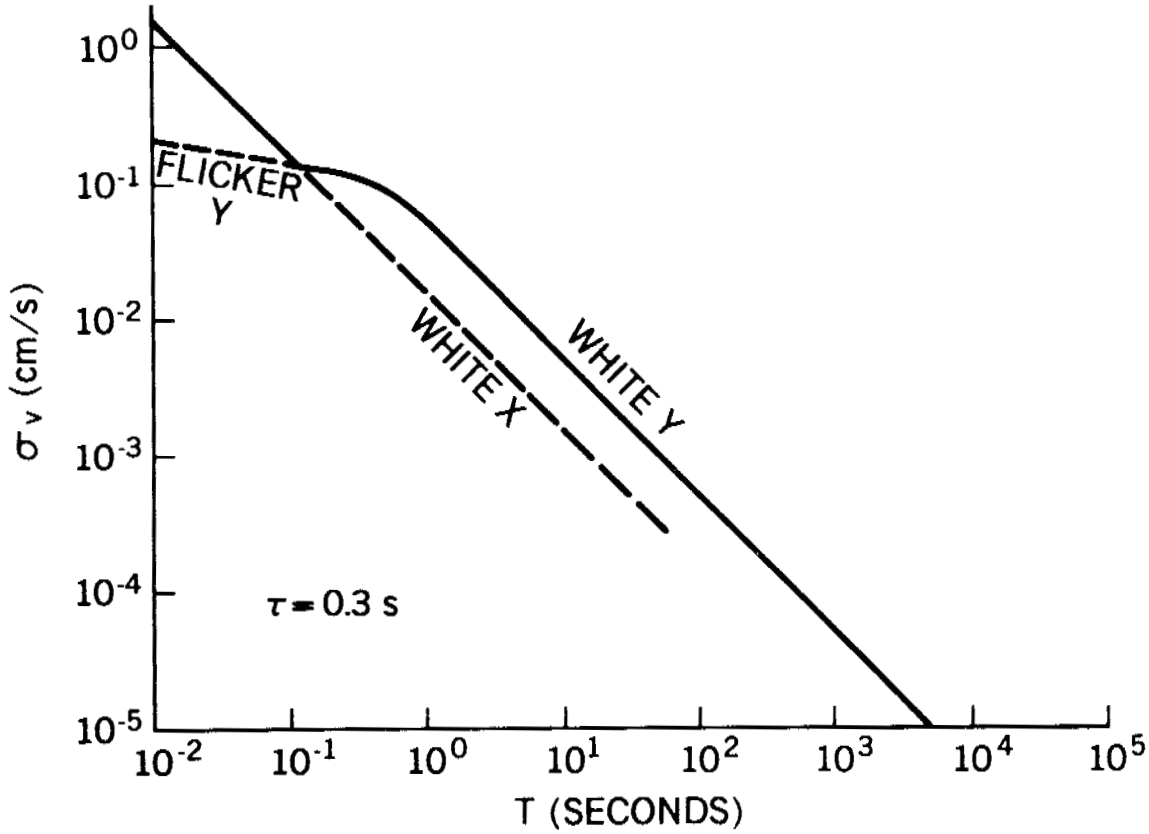

Fig. 8—σ_v(T, 0.3 s) for Hewlett-Packard High Performance Cesium Standard

The last device we will consider is NASA's latest generation hydrogen maser. The $\sigma_y(\tau)$ curve is shown in Figure 9. The device uses an Oscilloquartz B-5400 as a local oscillator which it phase locks to the maser with a 1 Hz loop. Using this fact and Figure 9, by indirectly using (11), we can calculate the white x noise contribution to σ_v^2 with a 1 KHz band width as the sum of the oscilloquartz contribution with a 1 KHz bandwidth and the maser's contribution with a 1 Hz bandwidth. In this case, the other noise processes make a negligible contribution. σ_v(T, 0.3 sec) is shown in Figure 10. Even though σ_v is determined by white x noise, σ_v cannot be appreciably reduced by reducing f_h; σ_v is determined principally by the maser white x noise in the 1 Hz loop bandwidth.

RANGE FROM RANGE RATE

Integrating f_D yields the integral of range rate: range. Since the averaging process involves integration, not dividing the information stored in the counter

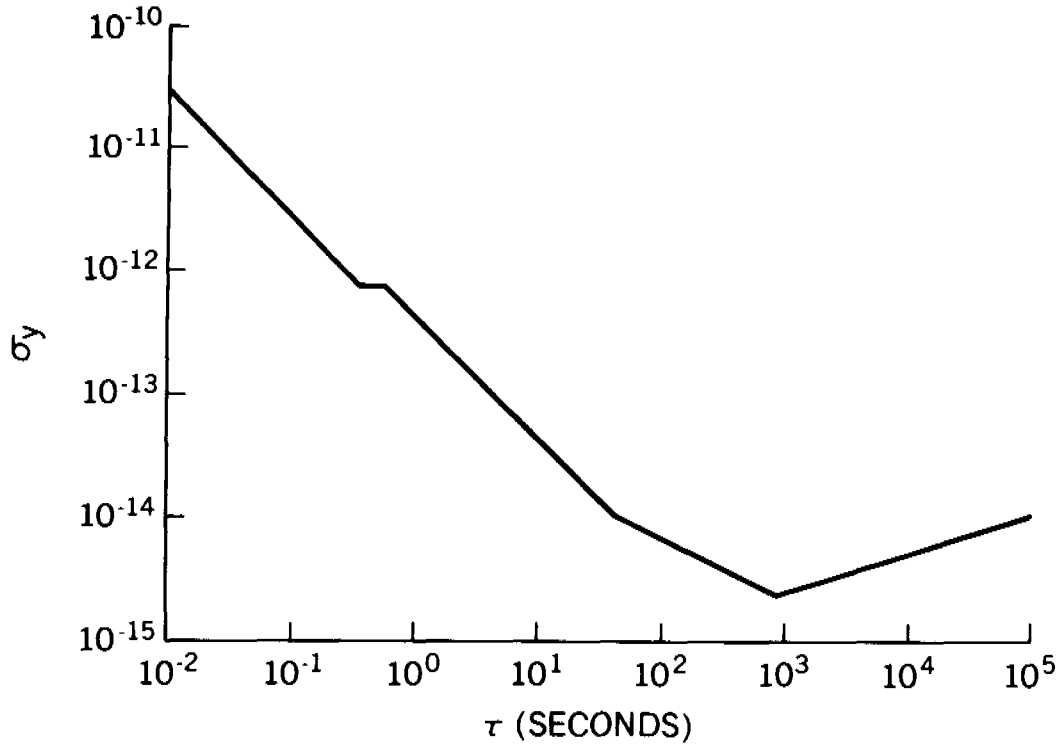

Fig. 9—$\sigma_y(\tau)$ for non-autotuned NASA hydrogen maser

register by T turns range rate data into range data. This means that the error in taking range data produced by the master oscillator is just:

$$\delta s(t, T, \tau) = T \delta v(t, T, \tau)$$

Similarly, this also means for random processes:

$$\sigma_s^2(T, \tau) = T^2 \sigma_v^2(T, \tau).$$

For the three sample oscillators, Figure 11 shows $\sigma_s(T, 0.3\ s)$. Notice again that taking non-destructive data instead of destructive data will yield better results in all cases but that of the crystal oscillator for T greater than 400 seconds.

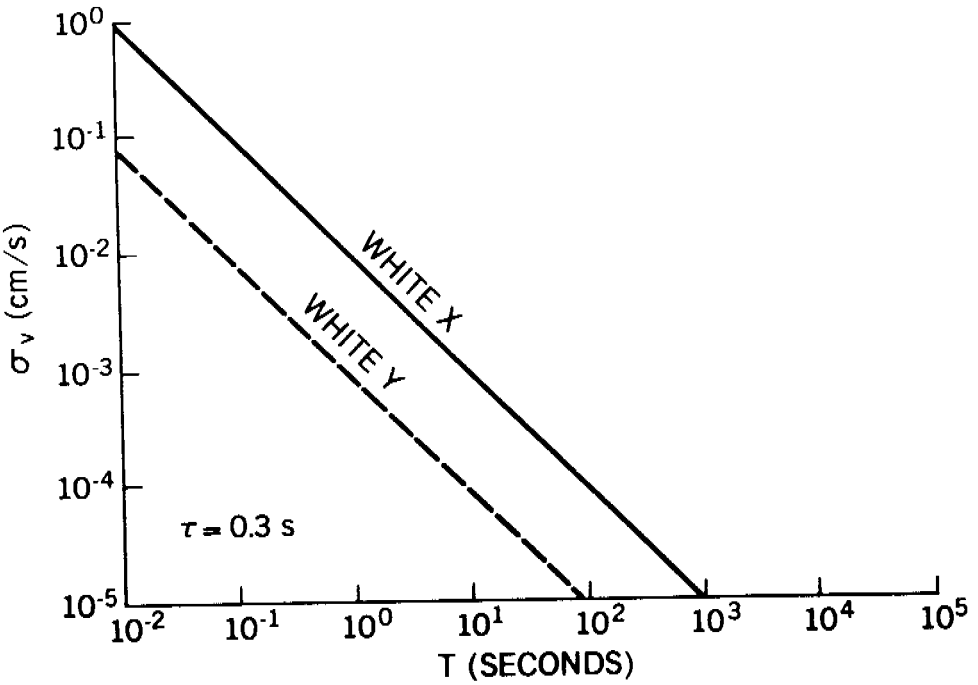

Fig. 10—σ_v(T, 0.3 s) for NASA hydrogen maser

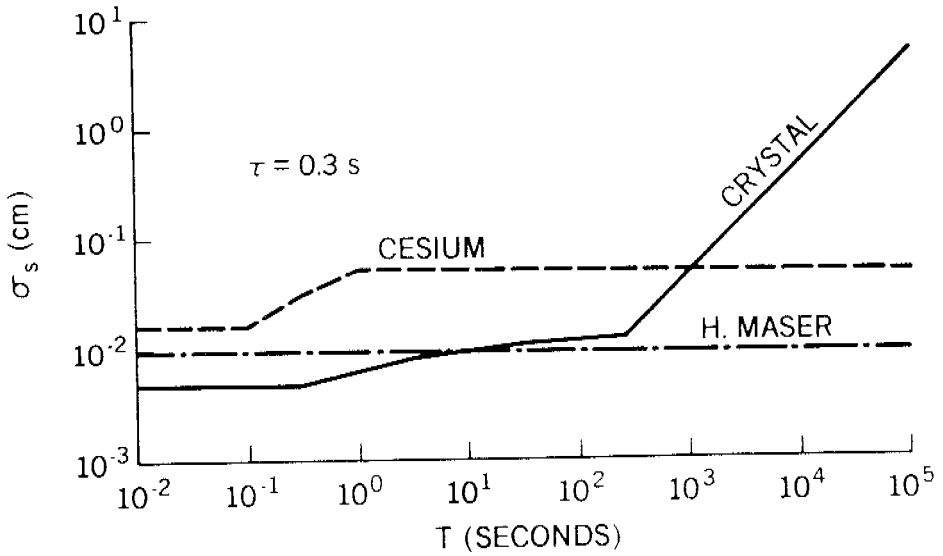

Fig. 11—σ_s(T, 0.3 s) for three master oscillators

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9. Both references 7 and 8 contain the relationships between $\sigma_y^2(\tau)$ and $\sigma_y^2(2, T, \tau)$. However, reference 7 has some errors which have been corrected in reference 8, so only reference 8 has been used to calculate the values given in Table 1.

10. Oscilloquartz S. A. CH-2002 Neuchatel 2, Suisse.

11. Hewlett-Packard Co. 1501 Page Mill Road, Palo Alto, California 94304.

QUESTION AND ANSWER PERIOD

MR. ALLAN:

One might suppose that the 60 hertz side bands will always plague us. One approach that we are going to take, haven't yet, is using for instruments that are displaced a long ways is optical fibers to connect the RF signals.

DR. REINHARDT:

Yes, I think that is a very good idea. I think that we are investigating that at NASA also, as a means for transmitting signals for longer distances.