DOWN TO EARTH RELATIVITY

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ABSTRACT

The basic concepts of the special and general theories of relativity are described. Simple examples are given to illustrate the effect of relativity on measurements of time and frequency in the near-earth environment.

Down to Earth Relativity

I. Introduction

As almost everybody knows, next March 14th is the 100th Anniversary of the birth of Albert Einstein, and so many celebrations and symposia are planned for 1979 that I fear all will become violently ill from an overdose of relativity well before mid year. For now, I would like to distill some of the salient aspects of both the special and the general theories of relativity and to relate them to clocks and frequency standards. After describing the basic concepts of special and general relativity, I'll discuss the size of the relativistic effects near the earth and the level of their experimental verification to indicate how well one might be able to rely on general relativity.

II. Special Relativity

Special relativity is partly concerned with the perceptions of observers viewing rods and clocks in uniform motion relative to one another (and not accelerating with respect to some "absolute" inertial frame which we won't worry about here). A key idea in Einstein's development of this theory involves the concept of simultaneity. If, as Newton assumed, there was a universal time coordinate that applied throughout all space, then there is no problem in our agreeing on a definition of the simultaneity of two events: We simply compare the readings of our "universal" clock. If the readings are the same at each event place, we agree that those events took place simultaneously.

If there is a spatial coincidence between two points, then there's again no problem agreeing on a definition of simultaneity because the points are co-located. We can use the same watch at the same place to see whether the events occur at the same time. That's no problem, with or without a universal time.

If there were spatial separation between two events, and if we could communicate between those two separate spatial points with infinite speed, then again, we'd all agree there would be no problem in deciding whether or not the events were simultaneous.

However, if we have spatial separation and the communication speed is limited by the speed of light, as Einstein thought, then, there is a problem. The definition of simultaneity is no longer intuitively obvious. In fact, as a simple, down to earth, example can show, even with a rea-

sonable definition, there is not necessarily agreement on simultaneity among observers moving relative to one another.

Now, let us define simultaneity for events at spatially separated points with communication between them possible via light signals. Concentrate for a moment on some given frame (Figure 1).

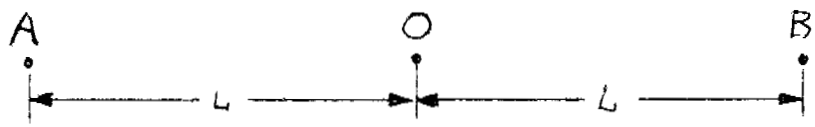

Figure 1
Any Single Frame

We're concerned about whether or not events that occur at points A and B in this frame are simultaneous. We can go to the midpoint between these two points, a distance L from A and B, and we can say that when an observer at point A sees an event he or she (hereafter "he" for economy) immediately transmits a light signal toward O and when the observer at point B sees an event he also immediately transmits a light signal to O. If these two light signals arrive at O simultaneously, then we say that the events A and B occurred simultaneously. That's a reasonable definition of simultaneity.

Now suppose we have two frames in relative motion. Consider, in particular, a down to earth example: the ground and a train (Figure 2).

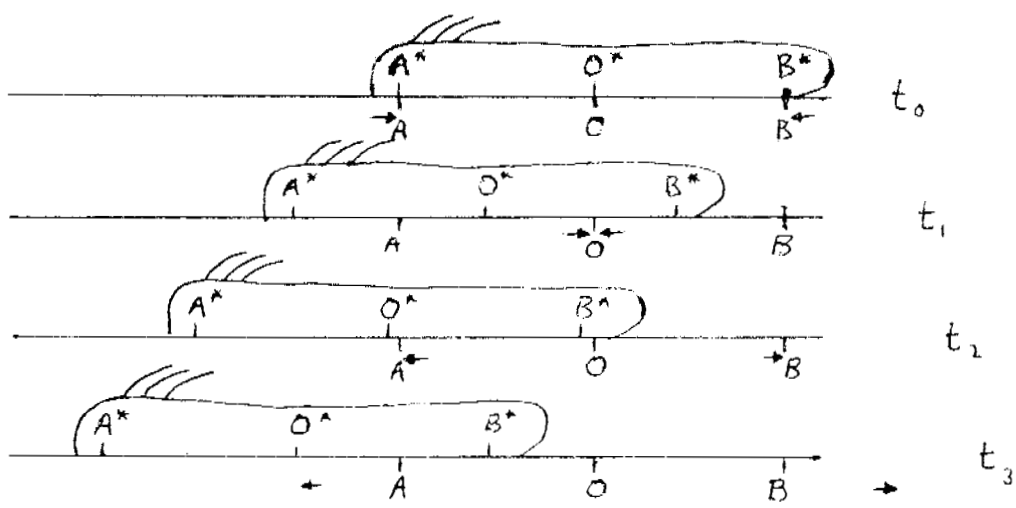

Figure 2
Two Frames In Relative Motion

(Any who know my flying habits, know that I have a special spot in my heart for trains even though they do tend to run a bit late.) Let us single out two points, A and B, on the ground, and set an observer halfway between. We follow the same procedure on the train so that at a certain instant, say t_o, we have our train point A* coincident with A, our observer O* coincident with O, and B* coincident with B. Suppose that an event occurs at both A and B at t_o as measured in the "ground" frame and that light signals are transmitted from A toward O, and from B toward O at that instant. As the light signals travel, the train is, of course, moving, say in the direction from B to A. Thus, the light signal from A is going to arrive at O* while the light signal from B is still travelling toward O*. A little while later, the two signals arrive at O, simultaneously, so the observer at O would say events A and B occurred simultaneously. But our observer at O* would not agree because he received the signals from A* and B* at different times. So one may conclude that if an event is simultaneous as measured according to our definition in one frame, the event will not necessarily be simultaneous as measured in another frame. Of course, there is nothing special about any one frame: the events could as well have been arranged to appear simultaneous to the observer at O* as to the observer at O.

In fact, one may conclude more generally that simultaneity is dependent on the motion of the observer. Einstein also thought that all observers moving uniformly with respect to one another should be equally valid observers so there should be no preferred (inertial) frame. Further, he felt that there should be no region of space-time singled out as more important than any other; he therefore assumed that space and time were homogeneous. This assumption implies that a linear transformation determines the relation of (Cartesian) space-time coordinates in one frame with respect to those in another. Einstein made one more assumption: the speed of light, c, is constant, such that the same value would be measured by any observer no matter what his state of relative motion. This seemingly provocative assumption had, of course, been upheld with exquisite accuracy in Michelson's and Morley's 1887 experiment.

To quantify these ideas, Einstein utilized a transformation which actually had been derived somewhat earlier, although with a different and inferior intellectual foundation, by Lorentz - the Lorentz Transformation:

$$\left.\begin{array}{ll} \underline{S^*} & \underline{S} \\ x^* = \dfrac{x - vt}{[1 - (v/c)^2]^{1/2}} & \\ t^* = \dfrac{t - (v/c^2)\, x}{[1 - (v/c)^2]^{1/2}} & \end{array}\right\} \qquad (1)$$

This transformation relates the coordinates $\underline{x}$ and $\underline{t}$, defined in a frame S, to the corresponding coordinates in a frame S*, where S* has a velocity $\underline{v}$ with respect to S. One can see that these transformations are linear in $\underline{x}$ and $\underline{t}$ and in x* and t*. Since neither S nor S* is "preferred", we should be able to invert the equations and obtain the same description, and, indeed, we do, except for the sign inversion of the velocity:

$$\left.\begin{array}{ll} \underline{S} & \underline{S^*} \\ x = \dfrac{x^* + vt^*}{[1 - (v/c)^2]^{1/2}} & \\ t = \dfrac{t^* + (v/c^2)\, x^*}{[1 - (v/c)^2]^{1/2}} & \end{array}\right\} \qquad (2)$$

Because S* has a velocity of $\underline{v}$ with respect to S, S has a velocity minus $\underline{v}$ with respect to S*. There is nothing in the transformation that singles out any one frame as special, despite the theory's being called "special" relativity (for a different reason, discussed below).

Let us now turn to the question of clocks. What in particular, does special relativity say about clock rates? If there is a clock at rest in the frame S* and one measures, from frame S, the interval between two "ticks", the result will be different from the corresponding measurement made by an observer in frame S*. The numerical relation between these measurements made in different frames is given by

$$\Delta t = \frac{\Delta t^*}{\sqrt{1-(v^2/c^2)}} \quad . \tag{3}$$

In other words, the observer in S thinks the interval between ticks is longer than does the observer in S*. The ratio of the two intervals is given by the Lorentz factor:

$$\sqrt{1 - (v^2/c^2)} \quad . \tag{4}$$

Similarly, since these are symmetric situations, if the clock had been in S, and the observer in S* were to measure the interval between two ticks, a similar relation would be obtained:

$$\Delta t^* = \frac{\Delta t}{\sqrt{1-(v^2/c^2)}} \quad . \tag{5}$$

The point is that, with the clock in S*, one is actually comparing it to a series of (identical) clocks distributed in S which the clock in S* passes as it moves with respect to S; similarly, with the clock in S: it moves with respect to the fixed (identical) clocks in S*. There is no paradox in the relationship being symmetric. One may conclude from this analysis that a clock always appears to run fastest in its own frame. When an observer is at rest with respect to the clock, he thinks the clock is running faster than when he is in uniform motion with respect to that clock. This effect has been verified very well in the measurement of the lifetime of unstable elementary particles. Such particles in cosmic rays, and in accelerators, often move with velocities v very close to c, and this lifetime enhancement factor can then be very large because the Lorentz factor tends to zero and its inverse to infinity. Studies of mu-mesons have verified this effect with very high accuracy.

III. General Relativity

The theory of relativity we just discussed is special in the sense of being restricted. It is silent on the subject of gravitation; it is concerned primarily with physics in (inertial) frames moving uniformly with respect to one another. Einstein felt that concern was not sufficient; he

wanted to introduce gravitation. Einstein was unhappy with Newton's theory of gravitation which had existed unchallenged for about two centuries. Newton's theory did, of course, have one small problem. There was a minute, but annoying, discrepancy between the observations and the theory which became noticeable in the late 1850's and was quite well established by the early 1900's. Einstein was not upset about Newton's theory only because of a mere disagreement with observations; his concern was a matter of principle.

Einstein did not accept Newton's theory because it implied action at a distance. In this theory, the force felt by body A due to body B depended on the location of body B at the very instant that body A felt the force. But if no signal can travel faster than the speed of light, how is body A to know where body B was located at that instant? This aspect was a severe drawback to Newton's theory in Einstein's mind and he set about the development of an alternative. The process took about a decade. The main principle upon which he based this general theory of relativity is the so-called "principle of equivalence".

One can state this principle in various ways. A usual way is to state that the effect of a gravitational field locally is indistinguishable from an inertial acceleration. The example usually given is that of an "Einstein elevator". Suppose a laboratory is enclosed in an opaque small elevator and placed in a gravitational field, such as on the surface of the earth. The scientists inside feel the force of gravity but cannot unequivocally identify it as such. They may do any physics experiments and obtain numerical results. However, suppose now the laboratory were taken away from the earth and accelerated uniformly with a rocket. If the scientists in the laboratory were to repeat all their experiments, the principle of equivalence states that they will get exactly the same numerical answers, provided that the inertial acceleration is exactly equal to the gravitational acceleration.

Another statement of the principle of equivalence can be given in terms of the ratio of gravitational to inertial mass. Gravitational mass is the mass that appears on the right hand side of the equation that expresses Newton's law of gravity:

$$F = \frac{Gm_g M_g}{r^2} \qquad (6)$$

where G is Newton's constant of gravitation, m_g is the (gravitational) mass of the body being acted upon, M_g is the (gravitational) mass of the body attracting m_g, and $\bar{r}$ is the distance between them. The inertial mass is the coefficient of the acceleration, a, in Newton's law of motion:

$$F = m_i a \tag{7}$$

The ratio of these two masses, according to the principle, is independent of the composition of the bodies and independent of the mass of the bodies. It is a universal constant. This principle, although not so named, was also accepted by Newton. In fact, he was the first to verify it quantitatively, achieving an accuracy of approximately 1 part in 1,000.

What can we infer from this principle of equivalence? One of the things Einstein inferred was that the trajectory of a particle could depend only on the geometry of space and time. By the principle, the trajectory did not depend on the particle itself, on its composition, or on its mass (except for the "back reaction" which I ignore here). It doesn't matter whether we have a pea, a flashlight, or whatever; it will move on the same trajectory because it will be affected in the same way as any other mass. Thus, Einstein reasoned that one could talk about the trajectories being merely a property of the geometry, and having nothing to do with the particular object that was moving along the path.

What determines the geometry? Einstein felt that the geometry should be determined solely by the mass, or, more precisely, the mass-energy, distribution in the universe. But isn't it contradictory to say that the path of the particle doesn't depend on the particle, only on the geometry, and that the geometry depends only on the mass distribution? Certainly the particle is part of the mass distribution. Yes, but if one considers the particle to have an infinitesimally small mass, it won't affect the geometry, and to that extent, these statements are consistent. But this "closed loop" aspect is a key to Einstein's theory.

Einstein may have been guided in developing his "field theory" for gravitation by analogy with Newtonian physics. In Newtonian physics, one obtains the gravitational potential from the mass distribution. In other words, the gravitational potential everywhere in space is determined by the mass distribution. In fact, the potential, Φ, is determined by Poisson's Equation:

$$\nabla^2\Phi = 4\pi\rho \quad , \tag{8}$$

where ρ is the mass density. Only the gravitational potential appears on the left side and only the mass (density) on the right side. This equation is a linear, second-order, partial differential equation for Φ. Einstein in effect generalized this purely spatial expression, to an analogous space-time expression that also allowed the geometry to be non-Euclidean. He used Riemannian geometry and developed an analogous equation where, on the right side, the mass density is replaced by the energy-momentum tensor:

$$G_{\mu\nu}(g_{\mu\nu}) = 8\pi T_{\mu\nu}; \quad \mu,\nu = 1 \to 4. \tag{9}$$

As in the Newtonian case, only the right side contains the "mass" terms; only the left side contains the "geometry" dependence. The geometry here is defined in a metric space. The so-called "metric tensor" $g_{\mu\nu}$ in essence expresses the "connection" between neighboring points in this space-time:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{10}$$

The interval ds is the "distance" between two infinitesimally separated points in the space-time. To evaluate ds^2, one sums over all values of the two indices μ and ν which run from 1 to 4 and correspond to the three spatial and the one temporal dimension. In Cartesian coordinates, in Euclidean three-dimensional space, $ds^2 = dx^2 + dy^2 + dz^2$; Equation (10) is the generalization for a Riemannian metric space.

Einstein made other assumptions, namely that this energy-momentum tensor is a conserved quantity in a sense analogous to the conservation of energy and momentum in Newtonian physics. Further, he limited the derivatives on the left side to second-order derivatives of $g_{\mu\nu}$, in analogy to the second-order derivatives on the left side of the Newtonian equation (8). With those assumptions, one can uniquely determine the left side up to a term proportional to the metric tensor. The coefficient of this term, the so-called cosmological constant, Einstein first took to be zero, a position he deviated from later when he thought the universe was static; still later, he greatly regretted this temporary deviation. (It is now generally assumed that the cosmological constant is zero.)

Because of symmetry ($T_{\mu\nu} = T_{\nu\mu}$), Equation (9) represents only 10 independent equations, not 16. These are Einstein's field equations which he used as the basis for calculations. In Newtonian Physics, the field equations were not enough. Equation (8) indicates how the gravitational potential can be determined, but it doesn't tell one how to calculate the paths of light rays and particles. In fact, Newton never said anything, as far as I know, about the effect of gravity on light rays. As for the effect of gravity on massive objects, Newton had a separate assumption, his equally well-known law of motion, given in Equation (7). In relativity, the corresponding equations are the equations for geodesics in four-dimensional space-time. A very intriguing aspect of the general relativistic formulation is that a separate assumption for the equations of motion does not seem to be needed; the equations of motion follow from the field equations themselves. The basic reason that makes this result possible, though by no means guaranteed, is that the field equations of general relativity are non-linear. The Newtonian field equation, by contrast, is linear. The terms hidden in $G_{\mu\nu}$ in Equation (9) are, in fact, non-linear expressions in terms of the metric tensor $g_{\mu\nu}$.

IV. Magnitude of Relativistic Effects

What of the magnitude of the relativistic effects we might expect? We know, as Einstein also knew, that Newtonian physics is a very good approximation, at least in our neighborhood. So the Newtonian equations must be, in some sense, the first approximation for the solution to the relativistic equations. Deviations from Newtonian physics appear in terms proportional to v^2/c^2 as we saw from the Lorentz Transformation; in the general theory of relativity, deviations appear in terms proportional to the factor, GM/c^2r. The quantity GM/c^2 has the dimensions of length and is often denoted by r_g and called the gravitational radius of the body. We can evaluate r_g near the sun, say, to determine the order of magnitude of the relativistic effects there that are due to gravitation. We find that, for the sun, $r_g \simeq 1.5$ km; by contrast, the radius of the sun is about 700,000 km. Thus, we can expect relativistic effects to appear at the level of two parts per million.

What about effects near the earth, which are of more direct concern for us? We find that the gravitational radius of the earth is near half a centimeter. In other words, the earth would have to be compressed down to half a centimeter before it would turn into a black hole. The radius of the earth is about 6×10^8 centimeters, so relativistic

effects near the surface of the earth could be expected to be on the order of eight parts per billion, not terribly large.

Let us now try to describe the relativistic effects quantitatively. To solve the field equations to determine the metric tensor $g_{\mu\nu}$, is no easy job. There are very few problems that have been formulated where $g_{\mu\nu}$ can be determined in closed form. The most famous one, solved by Schwarzschild very shortly after Einstein published his theory, is for a spherically-symmetric, static mass distribution. The solution exterior to that mass can be written as:

$$ds^2 = -\left(1 - 2\alpha\frac{GM}{rc^2} + 2\beta\left(\frac{GM}{rc^2}\right)^2 + \ldots\right)c^2dt^2$$

$$+\left(1 + 2\gamma\frac{GM}{rc^2} + \ldots\right)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2), \quad (11)$$

where, as stated above, ds^2 is the infinitesimal space-time interval and where the non-vanishing components of the metric tensor are the coefficients of dt^2, dr^2, etc. These coefficients, as here, are often written as a power series in GM/c^2r. In general relativity, the parameters, α, β, and γ, don't appear; they are identically one. The higher-order terms, indicated by "...", do not appear either; they are identically zero in general relativity. The reason for writing the metric in this "generalized" form is to facilitate the testing of the theory. By a least-squares match of the predictions of the theory to the results of observations made, say, in the solar system, one can estimate the values of these parameters. If the estimates turn out to be unity to within experimental uncertainty, we conclude that the observations are consistent with general relativity. If they aren't, then general relativity is in trouble.

Given the metric tensor and the equations of motion, one can then calculate explicitly the paths of test particles and light signals. The calculations are a bit intricate; one cannot in general obtain "closed-form" solutions. One often uses a perturbation expansion in powers of r_o/r where the first approximation represents the Newtonian solution and the next higher approximation, the so-called post-Newtonian solution.

V. Simple Examples of Relativistic Effects on Frequency and Time

Let us, finally, turn to the predicted relativistic effects on the frequency of light signals and on clocks. We will treat first a very simple example to show how one can use elementary reasoning to obtain an answer, without employing the full armamentarium of general relativity. We'll need only to apply the principle of equivalence. Thus, suppose we have a transmitter and a receiver that are stationary but separated. Let the receiver, or observer, be on the surface of the earth; let the transmitter, at an altitude H above the observer, transmit a signal with frequency f (see Figure 3). The question is, "What frequency does the observer measure?"

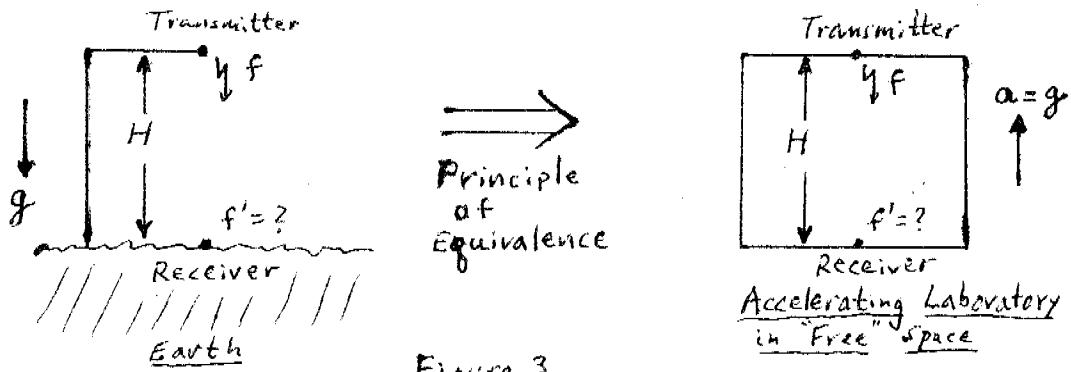

Figure 3
Application of Principle of Equivalence

An easy way to answer this question is to use the principle of equivalence. The system, or laboratory, we set up is equivalent to another where we replace gravity by an acceleration: We accelerate the laboratory at a value a, equal in magnitude to the acceleration g of the earth's gravity. We keep the observer and transmitter separated by the same distance H. At some instant, the transmitter sends a signal which the observer receives a short time, Δt, later. Let the velocity of the observer, at the instant of reception of the signal, relative to his and the transmitter's velocity, at the instant of transmission of the signal, be Δv. The value of Δv will be equal to the acceleration of the laboratory multiplied by the time interval between transmission and reception. Thus, using the principle of equivalence,

$$\Delta v = a\Delta t = g\left(\frac{H}{c}\right) \tag{12}$$

where Δt is just the time taken by light to travel the distance H ($\Delta t = H/c$), and where $a = g$. These are all approximate relations, valid to the first order in the small quantities. The frequency shift, $\Delta f \equiv f' - f$, of the observed frequency relative to the transmitted frequency, $\underline{f}$, is like a first-order Doppler shift and is given by

$$\frac{\Delta f}{f} \simeq \frac{\Delta v}{c} = \frac{gH}{c^2} \tag{13}$$

where we have substituted from Equation (12). This change in frequency represents, in fact, a violet shift.

Thus, the transmitter, at altitude H, sends a signal at frequency $\underline{f}$ and the observer receives a signal with a frequency greater by Δf. We note that the change in gravitational potential between transmitter and receiver is just the change in $-GM_\oplus/R_\oplus$, the gravitational potential for the earth:

$$\Delta\Phi = \Delta\left(-\frac{GM_\oplus}{R_\oplus}\right) \simeq \left(\frac{GM_\oplus}{R_\oplus^2}\right)\Delta R_\oplus \simeq gH \tag{14}$$

where $M_\oplus$ and $R_\oplus$ are the mass and radius, respectively, of the earth; and $\Delta R_\oplus$ is equal to H. The fractional change, $\Delta f/f$, in frequency and the accumulated difference, $\Delta\tau$, in apparent clock readings after elapsed time τ are given by:

$$\left.\begin{aligned} \frac{\Delta f}{f} &\simeq \frac{\Delta\Phi}{c^2} \\ \Delta\tau &\simeq -\frac{\Delta\Phi}{c^2}\,\tau \end{aligned}\right\} \tag{15}$$

In other words, if the observer had a clock identical in construction to that governing the transmitter, and if the observer knew the value of the transmitted frequency, as determined at the transmitter, by the clock there, the observer would infer that his clock was losing time relative to the clock in the lower gravitational potential of the transmitter. Of course, this "relativistic" loss can easily be taken into account in any comparison.

Let us consider another example. Suppose a frequency standard were in a circular orbit about the earth, and sup-

pose, incorrectly, that the first-order Doppler shift and the earth's rotation were negligible. Suppose, further, that a signal of frequency, f_s, is transmitted by the satellite and received on earth, and that the frequency of that signal is measured on earth, with equipment governed by a clock identical in construction to the clock in orbit that governed the transmission of the signal. Under our assumptions, the frequency, f_e, measured on earth will be related to f_s by:

$$f_e = \frac{[1 + (2\Phi_e/c^2)]^{1/2}}{[1 + (2\Phi_s/c^2) - (v_s/c)^2]^{1/2}} f_s \quad , \tag{16}$$

where the subscripts s and e refer to conditions at the satellite and on the earth, respectively. Thus, the difference, $f_e - f_s$, in frequency is determined by the motions of the bodies and by their gravitational potentials. Recall from Equation (2) that for the motion itself, we have the factor $(1 - v^2/c^2)^{+1/2}$; but here, where we are considering frequency rather than time, this factor enters with the plus half power rather than with the minus half power. As we saw in the first example, although not in this more exact form, the gravitational potential also affects the frequency; the effect was linear in (Φ_s/c^2) with a coefficient of unity. This result can be recovered here, for $(\Phi_s/c^2) \ll 1$, by expansion of $(1 + 2\Phi_s)^{1/2}$. Since (Φ_s/c^2), (Φ_e/c^2), and (v_s^2/c^2) are all small near the earth, we expand the right side of Equation (16), rearrange, and obtain (with the aid of conservation of energy):

$$\frac{\Delta f}{f} \equiv \frac{f_e - f_s}{f_s} \simeq \frac{3GM_\oplus}{2(R_\oplus + H)} - \frac{GM_\oplus}{R_\oplus}$$

$$\simeq 3.5 \times 10^{-10} \left[\frac{3R_\oplus}{(R_\oplus + H)} - 2 \right] \quad , \tag{17}$$

where H is the altitude of the satellite. The ratio $\Delta f/f$, the apparent fractional change in frequency measured by the observer on the surface of the earth is thus of the order of a few parts in 10^{10}, where for H less than half the radius of the earth we observe a violet shift, and for H greater than half the radius of the earth we observe a red shift. Above half an earth radius, the effect of the motion dominates over the effect of the gravitational potential, and vice versa, below half an earth radius. With the combination of the motion and the gravitational potential

effects, we would measure either a violet shift or a red shift, depending simply on the altitude of the satellite. Were we to observe from a lower potential, that is, from a position higher above the earth than the satellite, we would measure a *red* shift. Remember, however, that this entire development must really be modified for the observer's motion and for the first-order Doppler shift, both of which were ignored in this example.

VI. Validity of the General Theory of Relativity

Now let us address briefly the question of whether or not general relativity is a valid theory. It is clear in principle that at some level general relativity must "break down", because it is incompatible with quantum mechanics. No one has yet been able to formulate a satisfactory quantum theory of gravity, although there are some good ideas currently being explored. As one makes observations on a more microscopic scale, quantum mechanics plays an increasingly important role. At what length scale will quantum gravity actually be important? One answer is based on the evaluation of the "fundamental" length that can be formed from the gravitational constant, the speed of light, and Planck's constant, $\hbar$, which is a measure of the importance of quantum phenomena. This length is called the Planck length and is given by:

$$L_P \simeq \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 1.6 \times 10^{-33}\,\mathrm{cm} \quad , \qquad (18)$$

where, in accordance with convention, the "slash" on $\hbar$ denotes division by 2π. It is clear for present PTTI purposes that one need not worry about such length scales. It will be a long time before anyone will conceive of practical experimental procedures that will expose what happens at these length scales. Quantum theories of gravity currently under study center on so-called "super gravity", which tries to unite general relativity and quantum mechanics in a "higher level" theory for which general relativity will be the appropriate macroscopic limit. Testing the validity of these ideas is hopelessly beyond present experimental capabilities.

In the macroscopic world of the solar system, relativistic effects are very small. In addition, they have been verified by measurements to one percent or better. The relativistic effects of motion and gravity on clock rates, in particular, have been verified to approximately one hundredth of one percent already. A relativistic effect on trajectories, the prediciton of a non-Newtonian advance of

the perihelion position of Mercury's orbit, has been verified to about half a percent. The predicted deflection of light rays, and the predicted increase in echo delays, have been verified to the order of one percent, and a few tenths of one percent, respectively.

There is no problem, in principle, in applying the general theory of relativity to the solar system, and, in particular, to the earth environment at a useful level of accuracy. The situation is all very well defined by the principles of the theory. Unfortunately, how to apply these principles is not always so clear to those who try. As one consequence, apparent paradoxes have appeared in the literature, as well as many other errors. But, at the level of accuracy of interest to PTTI, these are the problems of those doing the calculations, and not the problems of the theory. The theory is quite reliable and often useful at this level of accuracy.

QUESTIONS AND ANSWERS

DR. CARROLL ALLEY, University of Maryland:

I think it is appropriate for this audience to realize that the first practical applications of Einstein's ideas in actual engineering situations are with us in the fact that clocks are now so stable that one must take these small effects into account in a variety of systems that are now undergoing development or are actually in use in comparing time worldwide.

It is no longer a matter of scientific interest and scientific application, but it has moved into the realm of engineering necessity. So talks like this are very important to try to acquaint the community with these fundamental principles, because the uncertainties have, indeed, arisen in lack of understanding of what is going on, rather than in the basic ideas.

DR. SHAPIRO:

Yes, in fact I left out one slide where I meant to show what the accumulated effect, say, in a day would be if you took two identical clocks, put one on the ground and one in the spacecraft in orbit around the earth at some nominal altitude.

Of course, we can cancel it out as we saw, but what would be the order of magnitude of the accumulated difference in the readings of the two clocks per day? And it is about 20 microseconds. So it can be quite substantial.

Of course, that is a little bit of a spoof since we don't yet have such extremely stable absolute standards, so if you put a clock in orbit and just measure its rate in orbit, then you would, in effect, automatically correct for these relativistic effects, provided it was a circular orbit and provided certain other things were true.

But when one gets down to the tens of nanoseconds level, and one worries about eccentric orbits and various other things, then it is true that these effects, small as they are, are not negligible compared to the accuracy that you can achieve with clocks.

The first really practical application that I know of that people are worried about is in the GPS system, where the effects are of the order of tens of nanoseconds for some of the applications.

DR. ALLEY:

For the GPS, albeit a 12 hour orbit, it is 38 microseconds per day.

DR. SHAPIRO:

That is true. But I say that you can get rid of that very easily by the redefinition of rate.

DR. ALLEY:

Yes.

DR. SHAPIRO:

But you still have to worry even in comparisons within a day of the order of tens of nanoseconds.

DR. ALLEY:

If I may be permitted one more comment: In the summer of '77 we actually carried out the Einstein falling-elevator experiments using the earth falling towards the sun. We transported clocks essentially from the floor to the ceiling by carrying them from the northern hemisphere to the southern hemisphere at the time of the summer solstice, when the axis is tilted toward the sun. We verified for clock rates that the potential of the sun does not effect the clock rates between floor and ceiling in the freely-falling elevator earth. Thank you.

DR. SHAPIRO:

There are many experiments, as I alluded to, that verified various aspects of general relativity. I felt I couldn't do justice to all of them, and therefore I did justice to none of them.

DR. CHARLES MARTIN, Defense Mapping Agency:

I would like to make one comment here because I think it's quite important in terms of our potential utilization of the global positioning system. I don't think there is any question about the microsecond errors if you do not take them into account.

But I think it is certainly important that we all realize that the capability, the theory, is adequate to take into account relativity errors to the level of, say, 20 or 30 nanoseconds.

DR. SHAPIRO:

No. To much better than that. My main message was the theory makes very specific predictions and they have been verified to a small fraction of one percent as far as clocks are concerned.

So, simply on the experimental verification level, you can believe them to the sub-nanosecond level. But as far as the theory is concerned, there is no good reason to believe it breaks down there just because you haven't tested below there. There is no theoretical reason that it should break down just below. And it does make very specific predictions. The problems arise, as I said, when people don't fully understand the theory when they try to use it in their calculations.

MR. ALLAN, National Bureau of Standards:

I again think for this audience, along the lines Professor Alley mentioned, that for the GPS user in the future, because the earth is

spinning, these effects become very significant. If you synchronize two clocks on the surface of the earth via portable clock and via satellite (by GPS), and ignore that the earth is spinning, assuming the Einstein synchronization technique, you can make errors of the order of hundreds of nanoseconds. So one has to be careful.

DR. SHAPIRO:

That is right. One has to be careful. But I am saying that the theory is very clear. I could work out any example, including the spinning earth, including flying clocks westward against the direction of earth (as was done already) and eastward with the direction. And there are differences there, because you are adding to or subtracting from the velocity of rotation of the earth. All of these things have been worried about and have been calculated and there is no problem, as long as you really understand the theory that you are applying.

MR. THOMAS MCCASKILL, Naval Research Laboratory:

We have a talk this afternoon in which we will present some results with the NTS satellites. In view of the high amount of interest that has been shown on the relativistic effects, we will bring a couple of slides that Mr. Buisson presented last year, which show the difference in frequency between a cesium clock measured on the ground and a cesium clock that was placed in orbit, which verified the first order relativistic effect.

DR. ALFRED KAHAN, Rome Air Development Center:

In your opinion, then, is there any experiment that still needs to be done to further prove the general theory of relativity with satellites, flying clocks? Or is the theory so good that we have confirmed to the one-percent or half-percent level that we don't need any more experiments?

DR. SHAPIRO:

I am a firm believer that physics is an experimental science and when one has the opportunity to test to a higher level of accuracy one should, provided it doesn't cost a major fraction of the gross national product.

And one has to draw some reasonable position there between do-able but hugely expensive and do-able but not such a great gain. I believe in experiments if you can make an order of magnitude gain in the experimental limit: It is worth a reasonable amount of money.

If you are going to make a ten percent gain, I personally wouldn't bother doing the experiment. There are some effects of general relativity that haven't been observed at all at any level that are important.

For example: The dragging of inertial frames due to the spinning of the massive body were predictions worked out from

general relativity as long ago as 1918. They have never been verified because the effects are very small.

There are several possible ways of getting an experimental handle on this with earth experiments, including flying spinning gyroscopes and so forth, but they are technically very difficult and very expensive to perform, and it is not clear yet that we are really ready to do that.

DR. ALLEY:

I would like to adopt a slightly different stance. The confusion in the understanding of the fundamental principles is widespread even among authorities.

I mean, there are recently published papers in the literature making predictions coming from people who should know better. For example, on this falling earth experiment I mentioned, one of the leading theorists in Europe in general relativity published in Physics Letters the flat statement that clocks would run at different rates at the North Pole and South Pole at the time of the solstices.

This is flat wrong, which he now admits. But there is a tremendous amount of intuition that is lacking in understanding general relativity, which we have in electricity and magnetism. And I would submit that the performance of clock experiments that we are now able to do will contribute vastly to developing this kind of intuition.

In a certain sense the clocks in gravitational fields are analogous to magnetic filings in magnetic fields. And it is quite important to do these experiments when one is able to do them.

DR. SHAPIRO:

I don't like to disagree with my colleague, but I find that I must disagree strongly with what Professor Alley just said. I find that no amount of experiment can really take people away from wrong notions. For example, the twin paradox has created fanatics in great numbers and no amount of experiments quells that at all.

As far as theoretical physicists like the one to whom Professor Alley alluded, and whom he didn't mention and whom I won't mention, he was perfectly well convinced that he had made an error simply on a theoretical basis. It didn't take an experiment to convince him that he made an error.

It was perfectly clear that he just didn't apply properly the relativistic principles. Many people, if they are reasonable, can be convinced by the theoretical arguments, and having exposed their wrong step, they admit it.

The non-fanatics will be convinced by the theory, and the fanatics won't be convinced by anything.