

ON ESTIMATING THE EFFECTS OF CLOCK INSTABILITY  
WITH FLICKER NOISE CHARACTERISTICS\*

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ABSTRACT

Clock instability is an error source to high-precision radio metric and radio interferometric observations. Under most circumstances, such observations take place over a period of time within which modern precision clocks reveal a fluctuation characterized by a flicker noise. An undesirable property of flicker noise is the correlation among all observations, with hopelessly complicated correlation coefficients. This complication prohibits one from treating flicker noise as random noise in covariance analysis estimating its effects. This paper introduces two alternative approaches. The first is that of generating a sequence of number simulating the flicker noise and then treating it as a systematic error. A scheme for flicker noise generation is given. The second approach is that of successive segmentation: A clock fluctuation is represented by  $2^N$  piecewise linear segments and then converted into a summation of  $N+1$  triangular pulse train functions (TPTF). The statistics of the clock instability are then formulated in terms of two-sample variances at  $N+1$  specified averaging times. The summation converges very rapidly that a value of  $N > 6$  is seldom necessary. An application to radio interferometric geodesy shows excellent agreement between the two approaches. Limitations to and the relative merits of the two approaches are discussed.

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## INTRODUCTION

The developments of advanced technology in radio metric observations for space navigation (Refs. 1, 2) and in radio interferometric observations (Refs. 3, 4) have been in a high gear during the last decade. Among the error sources limiting the precision of these observations is the instability in time and frequency standards. The estimation of the effects of such clock instability is becoming a vital part in system designs.

For a short averaging time,  $\tau$ , most precision atomic clocks possess a two-sample variance (Allan variance) of frequency fluctuation,  $\sigma_y^2$ , which decreases as  $\tau^{-2}$  or  $\tau^{-1}$ . In other words, the fluctuation behaves either as a white phase noise or as a white frequency noise. When one estimates the effects of such clock instability via a covariance analysis with phase (or delay) observables, a white phase noise can be treated as a random process with no correlation among observations. A white frequency noise can be treated in the same way except that all observations are correlated, with simple correlation coefficients. Hence such clock instabilities can be handled without difficulties.

However, most radio metric and radio interferometric observations take place over a longer period of time within which a clock fluctuation reaches its "flicker floor", having a constant two-sample variance over all averaging times of interest. Such flicker noise has the undesirable characteristic of complicated correlation among all observations, especially when observations are taken at uneven intervals of time. Direct covariance analysis is generally impractical.

This paper introduces two alternative approaches. In the first approach, a sequence of numbers simulating flicker noise is generated over the time period of interest. Its effects can then be treated as that of a systematic error, with the error sensitivity defined by the sequence of numbers. The second approach is that of a successive segmentation: The statistics of a clock fluctuation are represented by the amplitudes of a number of triangular pulse train functions (TPTF). These amplitudes are, in turn, related to the two-sample variances at successively halved averaging times. The problem is thus reduced to that of estimating the effects of a number of (usually not more than 7) TPTF with specific RMS amplitudes.

Examples are given for an application to baseline vector determination by radio interferometry. The numerical results show excellent agreement between the two approaches. Limitations to and the relative merits of these two approaches are discussed.

CLOCK INSTABILITY CHARACTERIZED BY  $\sigma_y^2 \propto \tau^{-2}$  and  $\sigma_y^2 \propto \tau^{-1}$

For convenience in investigating the effects of clock instability, we shall briefly review the basis of covariance analysis. Let there be  $M$  independent phase observations from which  $N_p$  parameters are to be estimated. The "computed covariance matrix" is given by

$$P_x = (A^T W A)^{-1} \quad (1)$$

where  $A$  is an  $M \times N_p$  sensitivity matrix with the  $(m,n)^{th}$  element being the partial derivative of the  $m^{th}$  observation with respect to the  $n^{th}$  estimated parameter;  $W$  is usually a diagonal weighting matrix with elements  $\epsilon_m^{-2}$  where  $\epsilon_m$  is the assumed RMS random error of the  $m^{th}$  observation. The diagonal elements of  $P_x$  in (1) are the variances of the estimated parameters due to the assumed random error in observations.

The effects of an error source different from the assumed random errors can be calculated by the following "consider covariance matrix" (Ref. 5)

$$P'_x = (P_x A^T W) P_c (P_x A^T W)^T \quad (2)$$

where  $P_c$  is the error covariance matrix of the  $M$  observations.

For a clock instability characterized by a two-sample variance  $\sigma_y^2(\tau) \propto \tau^{-2}$ , its effects on the M observations are the same as that of an uncorrelated random error with a standard deviation  $\tau\sigma_y(\tau)$ . Thus  $P_c$  in (2) becomes a unity matrix multiplied by a constant  $\tau^2\sigma_y^2(\tau)$ . The effects on the estimated parameters are given by the square roots of the diagonal elements of  $P'_x$ .

For a clock instability characterized by a two-sample variance  $\sigma_y^2(\tau) \propto \tau^{-1}$ , its effects on the  $m^{\text{th}}$  phase observation is the accumulation of phase, from the beginning of the experiment, due to a white frequency noise. That is, the error covariance matrix  $P_c$  will have elements

$$\begin{aligned}
 p_{m,n} &= \tau\sigma_y^2(\tau) \sum_{i=1}^{\min(m,n)} (t_i - t_{i-1}) & (3) \\
 &= \tau\sigma_y^2(\tau) [t_{\min(m,n)} - t_0]
 \end{aligned}$$

where

$$\min(m,n) = \begin{cases} m, & \text{if } m \leq n \\ n, & \text{if } n \leq m \end{cases} \quad (4)$$

$t_0$  is the epoch of the experiment and  $t_i$  is the mean time at which the  $i^{\text{th}}$  observation is taken. With  $P_c$  calculated by (3) and (4), the effects of such clock instability on the estimated parameters are again given by the square roots of the diagonal elements of  $P'_x$  in (2).

#### SIMULATION OF CLOCK INSTABILITY CHARACTERIZED BY $\sigma_y^2 \propto \tau^0$

A clock fluctuation characterized by a constant two-sample variance independent of averaging time is said to behave as a flicker noise. It has strong correlation among observations. To the knowledge of the author no exact expression exists for the error covariance of observations due to such noise. A close approximation can be derived from the following flicker noise model of Barnes and Allan (Ref. 6):

$$\phi_m = \sum_{i=1}^m (m+1-i)^{2/3} g_i \quad (5)$$

This model simulates equally spaced (in time), discrete flicker noise from a sequence of independent, random numbers  $g_i$  (or discrete white phase noise) of unity variance. For observations taken at even time intervals  $\Delta t$ , the error covariance matrix  $P_c$  will have elements

$$\begin{aligned} p_{m,n} &= (\Delta t)^2 \sigma_y^2 \langle \phi_m \phi_n \rangle \\ &= (\Delta t)^2 \sigma_y^2 \sum_{i=1}^{\min(m,n)} (m+1-i)^{2/3} (n+1-i)^{2/3} \end{aligned} \quad (6)$$

where  $\langle \rangle$  denotes the ensemble average and  $\min(m,n)$  is defined in (4). For observations taken at uneven time intervals  $I_m \Delta t$ , the elements of matrix  $P_c$  become

$$p_{m,n} = (\Delta t)^2 \sigma_y^2 \sum_{i=1}^{\min(k_m, k_n)} (k_m+1-i)^{2/3} (k_n+1-i)^{2/3} \quad (7)$$

$$\text{with } k_m = \sum_{j=1}^m I_j \quad (7a)$$

Therefore, the calculation of the elements of  $P_c$  becomes complicated and impractical.

An alternative approach is to simulate the clock instability in the time period of interest by a flicker noise model, such as that given in (5), and then treat it as if it were a systematic error. (However, statistical results can be obtained only through averaging an ensemble of such errors. This will be further discussed later). In other words, the  $P_c$  in (2) is decomposed into  $CC^T$  where  $C$  is a column matrix with its  $M$  elements given by

$$\epsilon_m = \Delta t \sigma_y \phi_{k_m} \quad (8)$$

for observations at uneven time intervals  $I_m \Delta t$ . Here  $\phi_{k_m}$  is defined in (5) and (7a).

Equation (8) suffers from the disadvantage of having to record a very large number of random samples  $g_i$  and to perform a very large number of summations. A computationally more efficient flicker noise model can be generated by the following empirical recurrence formula (for  $\sigma_y = 1$ ):

$$\begin{aligned} \phi_m &= 1.95 \phi_{m-1} - 0.95 \phi_{m-2} \\ &+ \sum_{i=1}^{100} i^{0.6} g_{m+1-i} - 1.95 \sum_{i=1}^{99} i^{0.6} g_{m-1-i} \\ &+ 0.95 \sum_{i=1}^{98} i^{0.6} g_{m-1-i} \end{aligned} \quad (9)$$

where  $g_i$  are now random numbers of standard deviation 1.35.

To compare the two flicker noise models, eq. (5) and eq. (9), 4001 samples are computed from each model. Two-sample variances are calculated and plotted in Fig. 1. The  $\sigma_y^2 \propto \tau^0$  behavior is verified for both models with the recurrence formula of (9) being slightly better. Note that for larger  $\tau$  the number of samples in the calculation of  $\sigma_y$  is smaller and the uncertainty of  $\sigma_y$  is larger. With the flicker noise model given in (9), the solution covariance matrix  $P'_x$  can be calculated from (2) with  $P_c = CC^T$ ; the elements of the column matrix  $C$  are

$$\epsilon_m = \Delta t \sigma_y \phi_{k_m} \quad (10)$$

with  $k_m$  defined by (7a).

#### SEGMENTATION OF CLOCK INSTABILITY

In this section, the effect of clock instability is studied by an alternative approach. It is clear that a clock instability characterized by  $\sigma_y^2(\tau) \propto \tau^\alpha$  with  $\alpha \geq -1$  will have a cumulative effect. Hence, a clock with instability behaving differently from a white phase noise will appear as clock drift. Such clock drift in the time period of interest, say  $0 < t < T$ , can be approximated by a piecewise linear representation as shown in Fig. 2 (a) and (b). The number of segments can be arbitrary; however, for the convenience of the following study, it is selected to be  $2^N$ . Fig. 2 (b) is a special case of  $N = 4$ .

The piecewise linear representation of a clock drift can further be decomposed into the summation of a sequence of  $N + 1$  triangular pulse train functions (TPTF),  $F_n$ :

$$\text{Drift}(t) = \sum_{n=0}^N F_n(b_{n,1}, b_{n,2}, \dots, b_{n,2^{n-1}}; t) \quad (11)$$

where  $F_n$  contains  $2^{n-1}$  triangular pulses of widths  $T/2^{n-1}$  and of heights  $b_{n,1}, b_{n,2}, \dots$ , and  $b_{n,2^{n-1}}$  (for  $n = 0$ ,  $F_n$  contains half a triangular pulse of height  $b_0$ ). Fig. 2 (c) displays the component terms of (11). Therefore, a given clock drift functions which is

approximated by  $2^N$  piecewise linear segments is uniquely defined by  $2^N$  heights of triangular pulses. Since these TPTF are independent of and uncorrelated with one another their total effect is the quadratic sum of individual effects. That is,

$$\text{Effect of Drift } (t) = \left[ \sum_{n=0}^N F_n^2 (b_{n,1}, \dots, b_{n,2^{n-1}}; t) \right]^{1/2} \quad (12)$$

The RMS values of  $b_{n,m}$  are related to the two-sample variances  $\sigma_y^2$  by the definition of  $\sigma_y^2$ :

$$\sigma_y^2 (T/2^n) = \frac{1}{2} \left\{ \frac{2b_{n,i}}{T/2^n} \right\}^2, \quad n \neq 0 \quad (13)$$

where  $\{ \}$  denotes the RMS value. Hence all triangular pulses in the same TPTF have the same RMS height which is directly related to  $\sigma_y^2$  at a specified averaging time:

$$\{b_{n,i}\} = \{b_n\} = (\sqrt{2}/2) (T/2^n) \sigma_y (T/2^n), \quad n \neq 0 \quad (14)$$

$$\text{For } n = 0, \{b_0\} = T \sigma_y (T) \quad (14a)$$

Therefore, the RMS effect of clock instability with known  $\sigma_y(\tau)$  for  $T/2^N < \tau < T$  can be represented by the superposition of TPTF of specific RMS heights. Only terms with  $T/2^n$  longer than the shortest time interval between observations need be included in (12). For instance, a 12-hour experiment with a minimum time interval of 10 minutes between observations requires only 7 TPTF, with the last term containing  $T/2^6$ . Furthermore, for  $\sigma_y^2 \propto \tau^0$  (flicker noise), eq. (12) converges very rapidly; neglecting all but the first three terms will result in an error of less than 1%.

In most radio metric observations for space navigation, the estimated parameters are the amplitudes of diurnal sinusoidal functions (Ref. 2). Hence, over a view period of 8 hours or shorter, the error signature can be divided into three categories: A bias error (approximating  $\cos x$  with small  $x$ ), a ramp error (approximating  $\sin x$  with small  $x$ ) and a random error. On the other hand, radio interferometric observations for clock synchronization, baseline vector determination, polar motion/UT1 determination, etc. are taken "randomly" on many different radio sources. It is such randomness that loosens the coupling between systematic error sources and estimated parameters. However, for such "random" observations, the error sources can also be divided into bias, ramp and random errors.

The RMS values of bias, ramp and random components of TPTF are calculated in the appendix. The values for the first 7 TPTF are summarized in Table I. Since these components are independent and uncorrelated the quadratic sum of their effects yields the effects of the TPTF. Also, as mentioned earlier, the TPTF are independent of and uncorrelated with one another. Hence, the magnitudes of errors of the same categories (bias, ramp and random) from all TPTF can be quadratically summed together, the effects of which are then individually estimated. Therefore, the estimation of clock instability effects is reduced to the estimations of the effects of a bias error, a ramp error and a random error (white phase noise) which are trivial.

#### NUMERICAL EXAMPLES

To illustrate and compare the two approaches introduced above, they are applied to a problem of baseline vector determination by radio interferometry. The baseline chosen is 300 km in length with its center at a latitude of  $35^\circ$  North. Both 6-hour (33-observation) and 8-hour (44-observation) experiments are studied. The observation sequences are parts of a 30-hour sequence observing 14 extragalactic radio sources. To reduce the effects of observation sequence, each of the 6-hour and 8-hour time periods scans through the 30-hour sequence and the mean error is calculated from all possible 6-hour or 8-hour periods. The error source considered is a clock instability with  $\Delta f/f = 10^{-14}$  for all  $\tau$  on each end of the baseline, thus  $\sigma_y(\tau) = \sqrt{2} \times 10^{-14}$ . A unity matrix is chosen as the weighting matrix,  $W$ . The quadratic sum of the baseline component errors is to be examined.

In practice, one or more clock parameters can be included in the estimated parameter list to reduce the effects of clock instability (Ref. 7). In the following, we shall study the problem under different circumstances: Estimating 3 baseline components alone, 3 baseline components and a clock offset, and all the preceding plus up to 8 equal segments of clock rate offset. It should be noted that when  $2^m$  segments of clock rate offset are to be estimated, the effects of  $F_n$  for  $n \leq m$  are to be excluded.

Table II summarizes the baseline solution sensitivities to a bias error, a ramp error and a white phase noise. These sensitivities are to be used in the segmentation approach: The effects of clock instability are to be determined by (i) calculating the RSS values of bias, ramp and random components, according to (14) and Table I, from all TPTF of concern, (ii) multiplying by the corresponding baseline error sensitivities in Table II, and (iii) quadratically summing these three error components.

Fig. 3 compares the effects of the clock instability on baseline solutions as estimated by simulation approach and by segmentation approach, for both the 6-hour and 8-hour experiments. Excellent agreement between the two approaches is seen.

#### DISCUSSION AND SUMMARY

Two different approaches have been introduced for the estimation of clock instability effects on radio metric and radio interferometric observations. The simulation approach is straightforward and can be applied to any type of problems; but it requires the simulation of a flicker noise. The statistical characteristics of a flicker noise can be attained only when a large number of samples are included. In the above examples, the 33 and 44 consecutive observations scan through a 30-hour sequence, resulting in, respectively, 128 and 117 different clock instability samples. Hence the mean values of the solution errors approach to their statistical values. Without such averaging the results fluctuate a good deal. Fig. 4 shows such fluctuation of a solution error from the 128 individual samples of 33-observation sequence.

The segmentation approach requires the estimation of the effects of a bias, a ramp and a random noise. The RSS magnitudes of these components from a few TPTF need to be calculated. However, this approach results in statistical values of clock instability effects without the need of averaging over many samples. Also, the segmentation approach is so versatile that clock instability with any shape of  $\sigma_y(\tau)$  can be treated since the variation of  $\sigma_y$  is explicitly accounted for (cf. equation 14) in the error estimation procedure.

When a problem with a diurnal variation has a view period exceeding 8 hours, bias and ramp together can no longer represent the systematic error signature. An additional error component, a single triangular pulse,  $F_1$ , with its mean value removed, needs to be considered. This is equivalent to adding a quadratic term into the small-argument approximation of a cosine function. With this modification, the RMS value of the random component for  $n=1$  in Table I is to be transferred to the new component. Of course, the random component from  $F_n$  with  $n > 1$  will still be needed.

## APPENDIX

### Calculation of RMS values of Bias, Ramp and Random Components of TPTF of Unit RMS Heights

#### Bias Component:

A sequence of triangular pulses of height +1 has a bias value of +1/2. Since  $F_0$  and  $F_1$  contain no more than one pulse their bias values are simply  $\pm 1/2$ . For  $n > 1$ , each pulse in  $F_n$  may have an independent sign (+ or -). Since  $F_n$  contains  $2^{n-1}$  triangular pulses, each of them has an RMS bias value of  $(1/2)(1/2^{n-1}) = 1/2^n$ . The RMS value of bias for all pulses in  $F_n$  is the quadratic sum of those of all  $2^{n-1}$  pulses. Hence

$$\begin{aligned} \{\text{bias}\} &= [(2^{n-1}) (1/2^n)^2]^{1/2} \\ &= 2^{-(n+1)/2} \end{aligned} \tag{A.1}$$

#### Ramp Component:

It is readily shown that  $F_0$ , containing one half of a triangular pulse, has a ramp component of RMS value  $1/2\sqrt{3}$  and that  $F_1$ , containing one pulse, has no ramp component. For  $F_n$  with  $n > 1$ , the triangular pulses can be grouped into symmetrical pairs (with respect to the center of the time period). A pair of pulses with the same signs (++ or --) does not contribute to ramp component. A pair with opposite signs (+- or -+) has a ramp component of heights  $\pm 3m/2^{2n-2}$  where  $m$  is the separation between the two pulses of the pair ( $m = 1, 3, \dots, 2^{n-1} - 1$ ). The RMS value of such ramp component is  $1/\sqrt{3}$  its height, i.e.,  $(3m/2^{2n-2})(1/\sqrt{3})$ . The probability of forming a pair of pulses with opposite signs is 1/2 (the other 1/2 for pulses with the same signs). Hence the RMS value of the ramp component of all  $2^{n-2}$  pairs of pulses in  $F_n$  is given by

$$\begin{aligned} \text{ramp} &= \left[ \sum_{m=1,3,\dots}^{2^{n-1}-1} \left(\frac{1}{2}\right) (3m/2^{2n-2})^2 (1/\sqrt{3})^2 \right]^{1/2} \\ &= \left[ \left(\frac{1}{2}\right) (3/2)^{4n-4} \sum_{m=1,3,\dots}^{2^{n-1}-1} m^2 \right]^{1/2} \end{aligned}$$

$$\begin{aligned}
&= [(3/2)^{4n-3} (2^{3n-4} - 2^{n-2}) / 3]^{1/2} \\
&= [2^{-(n+1)} - 2^{-(3n-1)}]^{1/2}
\end{aligned}
\tag{A.2}$$

Random Component:

The total RMS value of  $F_n$  is the same as that of a single triangular pulse,  $1/\sqrt{3}$ . With the bias and ramp component given by (A.1) and (A.2) the RMS value of the random component of  $F_n$  is simply

$$\{\text{Random}\} = (1/3 - \{\text{bias}\}^2 - \{\text{ramp}\}^2)^{1/2}
\tag{A.3}$$

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TABLE 1

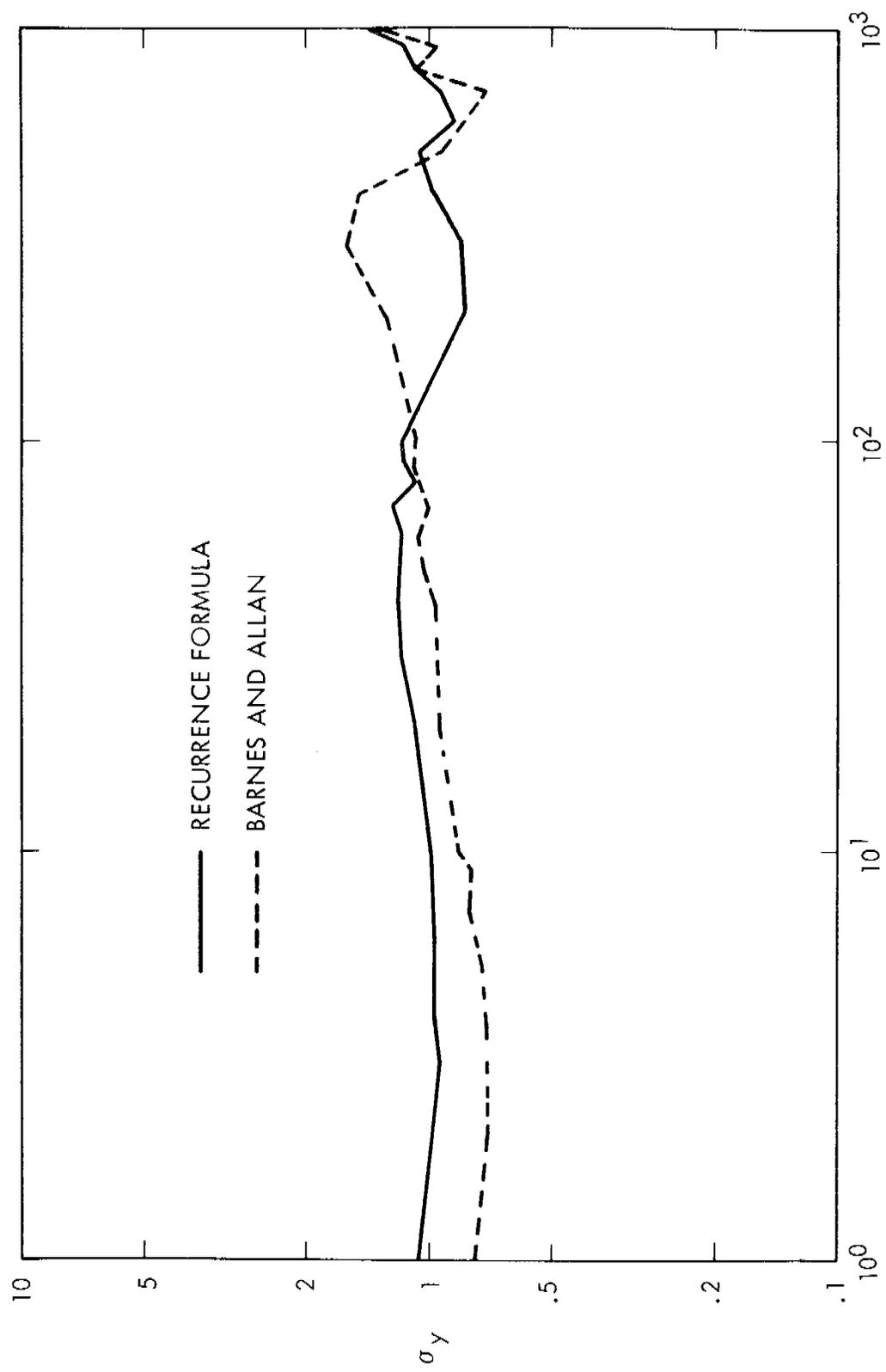
RMS Values of Bias, Ramp and Random Components  
of TPTF of Unit RMS Heights\*

n	{Bias}	{Ramp}	{Random}
0	0.500	0.289	0
1	0.500	0	0.289
2	0.354	0.306	0.339
3	0.250	0.242	0.461
4	0.177	0.175	0.521
5	0.125	0.125	0.550
6	0.088	0.088	0.564

\* A triangular pulse of unit height has a total RMS value of  $1/\sqrt{3}$ .

TABLE II: Baseline Solution Sensitivity to Observation Errors of Unit RMS Values

T	M	Error Type	$N_p = 3$	$N_p = 4$	$N_p = 5$	$N_p = 6$	$N_p = 8$	$N_p = 12$
6	33	Bias	1.32	0	0	0	0	0
6	33	Ramp	0.55	0.91	0	0	0	0
6	33	Random	0.60	1.05	1.07	1.09	1.11	1.18
8	44	Bias	1.33	0	0	0	0	0
8	44	Ramp	0.48	0.76	0	0	0	0
8	44	Random	0.50	0.89	0.90	0.91	0.93	0.96



AVERAGING TIME  $\tau$

FIGURE 1. TWO-SAMPLE DEVIATIONS OF TWO FLICKER NOISE MODELS

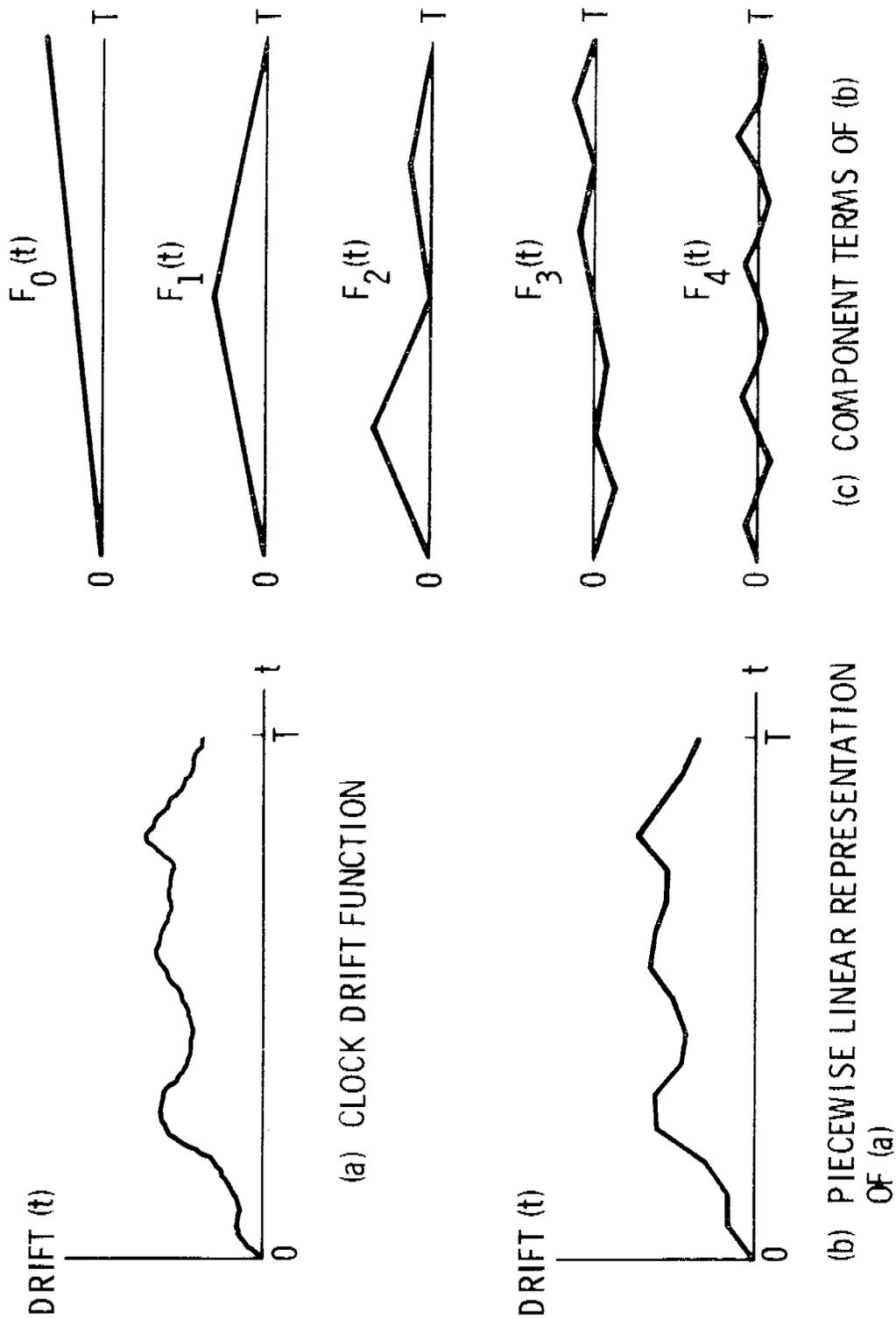


FIGURE 2. SEGMENTATION OF CLOCK INSTABILITY

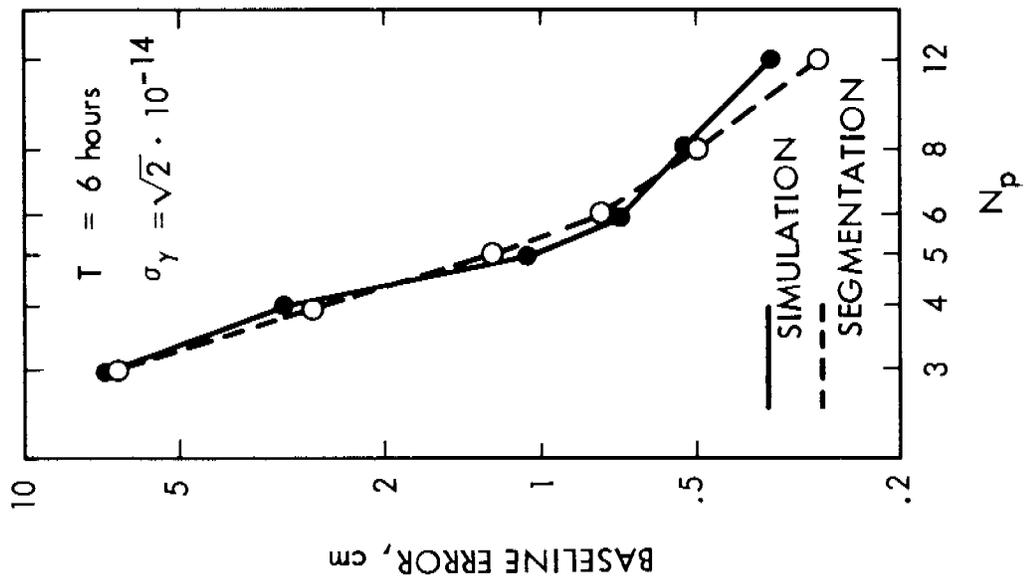
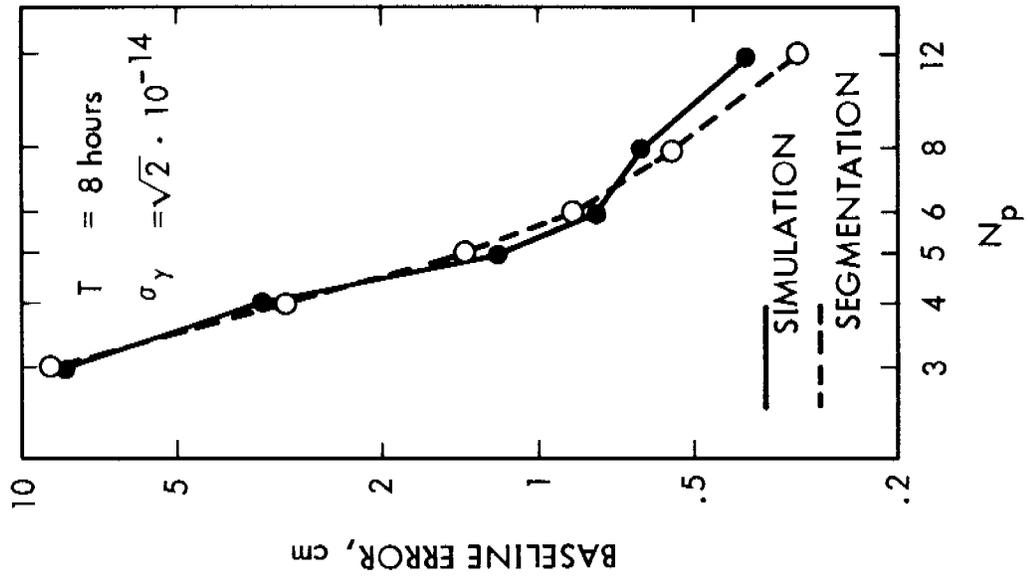


FIGURE 3. INTERFEROMETRIC BASELINE DETERMINATION ERRORS DUE TO CLOCK INSTABILITY

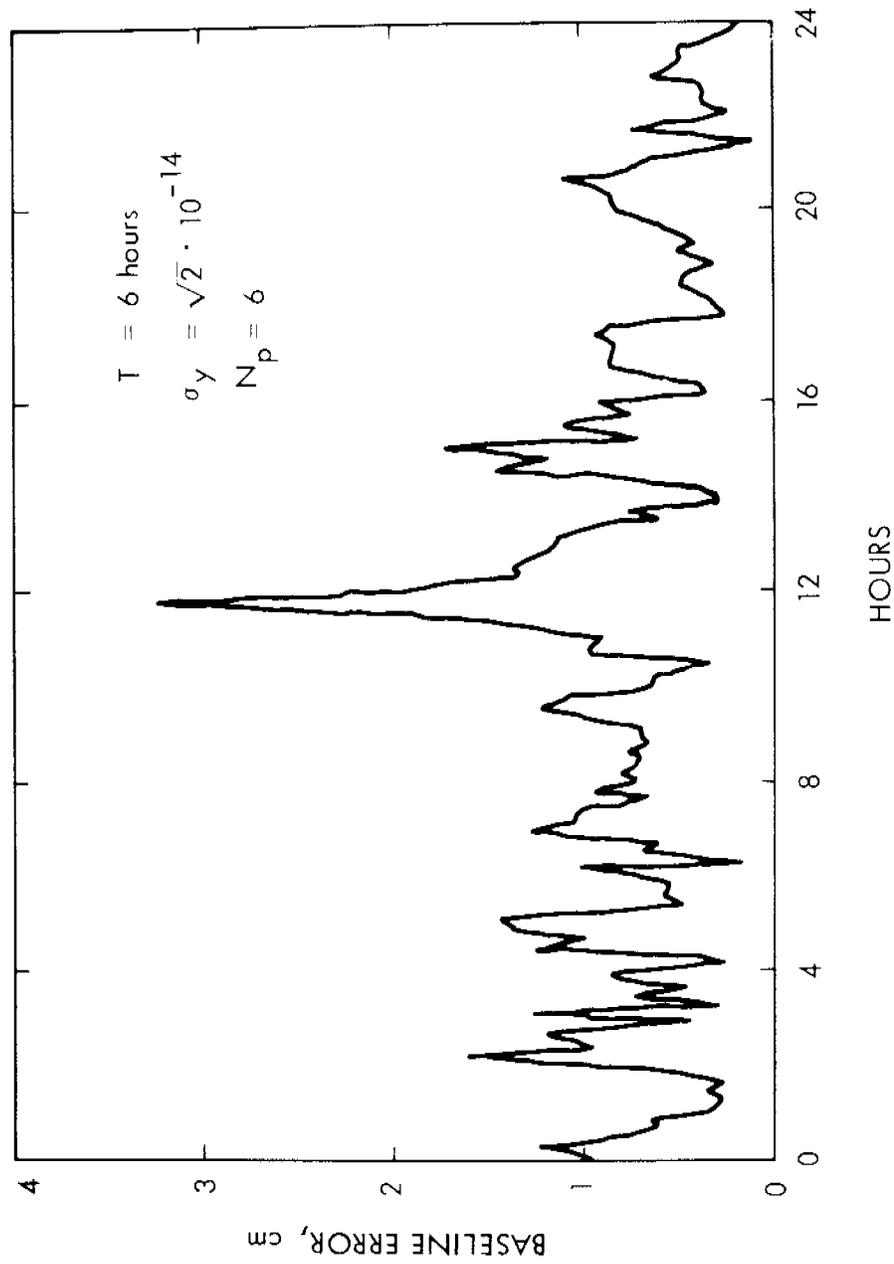


FIGURE 4. FLUCTUATION OF BASELINE ERROR ESTIMATED BY SIMULATION APPROACH

## QUESTIONS AND ANSWERS

DR. REINHARDT:

Recent data have indicated over the past couple years that the problem with hydrogen masers or with cesiums too is not really flicker noise. It is a combination of environmental effects and random walk which can be analytically treated much more easily than flicker noise. Have you looked into using those approaches to handling problems of correlation noise in frequency standards?

DR. WU:

Well, of course, flicker noise is just an assumption of clock instability. It is an approximation, but the segmentation approach I was just introducing here can be applied to any sigma versus tau shift. That means it doesn't necessarily have to be a flicker frequency noise.

DR. REINHARDT:

If you use other noise models which are really applicable like random walk of frequency or an environmental effect, they are analytically solvable, while flicker noise presents a lot of computational problems; they do not. And you might get a more fruitful result by using the real models for behavior.

DR. WU:

If the characteristic is purely white phase noise or white frequency noise, you can easily do it with conventional covariance analysis, but whenever there is some combination of these noises or you have some variations in the sigma versus tau shape, then you have difficulty.

DR. PETER KARTASCHOFF, Swiss Post Office

I wonder if in the segmentation approach you used here, what is the statistical uncertainty on these ramp and drift and random components in this approach, because I am asking myself if you just have a statistical uncertainty, if you repeat this process which will be of the same order of magnitude, then the uncertainty you get with the simulation process is mainly according to the theory of Audoin and Lesage?

Whereas, that there might be a danger that the segmentation approach used on limited data would give a too optimistic result, and then if you repeat the same experiment more and more, and

finally you get this very slow convergence to the flicker noise process, and there I would recommend what Victor Reinhardt said, that actually there have been for ten or more years, efforts to turn around the flicker process.

And the combinations of white and random walk rates and so on, but I think we have to live with the fact that the flicker process is here and nature doesn't care about mathematical difficulties; nature is there and the flicker process is there, and for many years we had no physical models on flicker process, but three years ago there was a conference in Tokyo only on flicker phenomenon.

Now, we have about ten models of flicker noise, ten physical models. We just do not yet know which is the good one, but I think we will have to live many years with the problem of flicker. I just would like to make these comments. One should look in this approach you made because it is very interesting in its computational simplicity, but I would say a little bit of warning. What is the real uncertainty? I would bet it will fall back on Audoin's and Lesage's prediction on uncertainty of the estimates. Thank you.

DR. WU:

Of course, what I am introducing here is just a technique to estimate clock instability with a given shape of sigma versus tau shift. So, whether the actual clocks will be right on flicker frequency noise or not is another story.