THE IMPLICATIONS OF PRECISE TIMEKEEPING FOR DOPPLER GRAVITATIONAL WAVE OBSERVATIONS

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ABSTRACT

Gravitational radiation from galactic and extragalactic astrophysical sources will induce spatial strains in the solar system, strains which can be measured directly by the Doppler radio link to distant spacecraft. We delineate current noise sources in Pioneer and Voyager Doppler data and make a comparison with expected signal levels from gravitational wave sources. The main conclusion is that it is possible to detect gravitational radiation with current DSN hydrogen maser systems stable in fractional frequency to $\pm 2 \times 10^{-14}$ over 1000 sec. In the future, however, a serious Doppler observational program in gravitational wave astronomy will require frequency systems stable to at least 10^{-16}, but at the same time the current single frequency S-band uplink transmission will have to be replaced by a dual frequency capability. In the meantime it is more likely that the S-band uplink will be replaced by a single X-band link, thereby improving the overall system frequency stability to the limit of the hydrogen maser system itself. This option, though attractive, seems more limited by the lack of X-band transponders on distant spacecraft than by the development of ground systems by the DSN. Earth tropospheric effects will not be a problem until stabilities of $\pm 5 \times 10^{-15}$ or better are realized.

INTRODUCTION

Gravitational radiation arises from the Einstein theory of gravitation (general relativity) which modifies the Newtonian concept of the gravitational force acting instantaneously at a distance to a modern view of a gravitational field which travels at finite speed c away from a source. The Einstein field equation, which is analogous to the Maxwell equations of electromagnetism (EM), is $G = 8\pi T$, where T is a second rank stress energy tensor representing the source of the gravitational field and G is a second rank tensor made up of

quantities describing the curvature of the four dimensional space-time continuum. Exact solutions of the Einstein field equations are few in number, and numerical techniques are now yielding most of the interesting descriptions of material interactions and associated Gravitational Waves (GW) [1]. Much of the physics of the generation of GW and their propagation can be understood by considering gravitation as a weak perturbation to an empty, flat space of special relativity. Under this restriction, it is possible to derive a wave equation from the Einstein field equations [2], and to predict gravitational radiation from material events in direct analogue to EM radiation from moving charges. However, unlike EM where both positive and negative charges exist, matter is made up of only positive mass, and as a result, the lowest order form of gravitational radiation is quadrupole, in contrast to the fundamental dipole EM radiation. Also, a spherically symmetric source of GW is impossible, and thus large deviations from spherical symmetry are required in sources useful for detection. For these reasons, in addition to the fact that energies of GW are about 10^{-43} times smaller than EM energies from a comparable source, laboratory experiments of the type performed by Hertz are practically impossible for gravitational radiation. Yet few theorists doubt the existence of GW, for the reason that once one has transformed gravitation from the Newtonian concept of action at a distance to a modern concept of disturbances in a gravitational field which propagate at a finite velocity, it is difficult to avoid the consequence that GW carry energy, and interact with matter. In fact, the existence of GW is more widely accepted than the continuing validity of general relativity.

If GW cannot be produced and detected in the laboratory, then we must look to strong natural sources. The coupling of gravitational waves to matter is weak, and only the most violent astrophysical events generate waves of sufficient amplitude for detection at earth. For example, the current Doppler gravitational radiation search with Pioneer 10 could marginally detect the waves from a collision of two black holes with a total mass of 10,000 times the mass of the sun at the distance of the center of the galaxy. However, a beneficial consequence of weak coupling is that gravitational radiation has an enormous penetrating capacity which would give astronomers a clear window onto parts of the Universe that are totally opaque to even the hardest X-rays, a view which would include the internal structure of supernovae and the details of gravitational collapse of objects with masses of 10^6 solar masses or more.

The technique of using Pioneer or other distant spacecraft to detect GW is to monitor the Doppler shift of the radio signal, continuously transmitted to the spacecraft and coherently transponded back to earth. If the velocity induced Doppler shift is removed from

the records, then the remaining data can be analyzed for GW. The characteristics of the GW signal, embedded in a Doppler time series, have been discussed previously [3] and will not be repeated here.

CURRENT EXPERIMENTS

We are currently using two spacecraft, Pioneer 10 and Pioneer 11, for the detection of GW. The first acquisition of GW data started on November 15, 1981, from Pioneer 10, and will continue until December 8, 1981. During this interval Pioneer 10 will be at opposition where the noise from interplanetary plasma scintillations is at a minimum and the chances for the detection of GW are greatest. About six months later, Pioneer 11 will be at opposition and data will be acquired again. We plan to follow this pattern for several oppositions, thereby obtaining three weeks of relatively low-noise data about every six months.

The uplink to Pioneer 10/11 is a 20 kW S-band (2.2 GHz) signal radiated from one of the 64-meter parabolic antennas of the Deep Space Network (DSN). The signal is tracked in a phase-locked loop on board the spacecraft and coherently transponded at S-band at a power of 8 watts. In normal DSN operation, the received signal is tracked in a phase-locked loop, a hydrogen maser clock being used to beat the frequency down to the Doppler tone.

The two important limiting noise sources on the Pioneer Doppler system are the weak signal levels at distances of 20 to 40 AU, and scintillations in the Doppler signal caused by scattering of the S-band signal by free electrons in the interplanetary medium. Our estimate for the Doppler noise in $\Delta f/f$ for the Pioneer spacecraft with its high gain 2.74m parabolic antenna fed by an 8 watt transmitter and using a 64m DSN receiving station is

$$\sigma_y \approx 7 \times 10^{-15} \left(\frac{100 \text{ sec}}{\tau}\right)\left(\frac{D}{5 \text{ AU}}\right)$$

where τ is the integration time for the Doppler signal, D is the distance of the spacecraft, and σ_y is the square-root Allan variance of $y \equiv \Delta f/f$. For a distance of 20 AU, and a 100 sec integration time, the noise in the Doppler link because of a weak signal is about 3×10^{-14}. Pioneer 11 will not exceed a distance of 20 AU until 1986, but Pioneer 10 is beyond that distance now and will reach nearly 40 AU by 1986. However, the noise in the Pioneer 10 Doppler link can be held to an acceptable level of 2 or 3×10^{-14} by increasing the integration time to 200 or 300 sec. Signal to noise limitations are not a serious problem for either spacecraft.

Another significant limiting error source for the Pioneer Doppler link is interplanetary phase scintillation associated with refractive

index fluctuations in the solar wind. Armstrong, Woo, and Estabrook [4] have reported observations of radio wave phase scintillation, using the Viking spacecraft. The phase power spectrum level varies by seven orders of magnitude as the Sun-Earth-spacecraft (elongation) angle changes from 1° to 175°. It is noteworthy that a broad minimum in the S-band (2.3 GHz) phase fluctuation occurs in the antisolar direction; the corresponding fractional frequency stability (square root Allan variance) is 6×10^{-14} for 1000s integration times. The ionospheric contribution is significant but it is dominated by the contribution from the interplanetary medium. Nondispersive tropospheric scintillation was not detected in the Viking data, and more recent work by Armstrong and Sramek [5], using data from the National Radio Astronomy Observatory's Very Large Array (VLA), indicates that tropospheric noise should not be evident in either the Viking or the Pioneer data.

In summary, it is realistic to expect a sensitivity of 6×10^{-14} in the Pioneer Doppler link at opposition, even under additional considerations of limits in the stability of the hydrogen maser frequency standard system, kT noise in the various electronic subsystems, nongravitational translational forces on the spinning spacecraft, and resolution limits in the Doppler extraction system.

FUTURE REQUIREMENTS FOR FREQUENCY STABILITY

We have shown in the previous section that current Doppler searches for GW are not limited by the DSN hydrogen maser systems, but instead by plasma noise in the S-band radio link. While Pioneer is equipped with only an S-band transponder, Viking and Voyager have both S-band and X-band on the down link in an 11:3 frequency ratio. This difference in frequency can be used to remove most of the plasma noise on the down link by making use of the dispersive nature of electron scattering [6]. Unfortunately, it is not possible to establish enough spatial and temporal coherency between the uplink and downlink to reduce the plasma noise significantly on the S-band uplink. We have learned from experience that the advantage of the Voyager radio system over the single frequency Pioneer system is that the plasma noise can be reduced by about a factor of two; there is one noisy S-band link on Voyager (uplink), and two noisy S-band links (uplink and downlink) on Pioneer. Therefore, while the best low-noise environment on Pioneer is at about 6×10^{-14}, on Voyager it is at about 3×10^{-14}, still slightly above the DSN hydrogen maser system. The radio system being integrated into the 1985 Galileo mission to Jupiter is essentially the same as Voyager, so there is no real prospect for improvement over current systems in the 1980's. One exception might be a mission to the Sun (Starprobe) in the late 1980's; it could be used for GW detection [7] because of its required flyby of Jupiter for a gravity assist.

The Doppler search for GW over the next decade will probably be carried out in the noise environment displayed in Fig. 1. Here, we plot the error in the Doppler frequency, expressed as square-root Allan Variance, as a function of the Doppler integration time for a number of significant noise sources. In the region of 1000 sec, a representative region for the GW search, the solar plasma noise clearly dominates. The dotted line is representative of current DSN hydrogen masers, although the present overall frequency system may be an order of magnitude worse. However, with sufficient effort, stabilities on the order of 10^{-15} over 1000 sec could be obtained.

We now address the problem of whether there are any GW sources of sufficient power to be detected in the noise given by Fig. 1. Estimates of the dimensionless amplitude of GW reaching the solar system from a variety of sources have been made by Thorne [8] and are shown in Fig. 3. The dimensionless amplitude represents the spatial strain in the gravitational field and is the quantity that is measured directly by the spacecraft Doppler technique. Thus at frequencies of GW in the VLF region of 10^{-4} Hz, we would not violate anyones "cherished beliefs" if we detected Doppler shifts $\Delta f/f$ of a few parts in 10^{13} from bursts of GW, but we would not expect to see bursts above a level of 10^{-15}. However, we might see a stochastic background of GW at a level of 10^{-14}, a level that is just a little beyond the reach of Pioneer and Voyager. A clear detection of GW could be achieved with bursts of unexpectedly large magnitude. In the absence of such bursts, we can report a limit on the magnitude of bursts hitting the solar system during the times when spacecraft are being used for detection purposes, and also we can place a limit on the stochastic background in the region of 10^{-4} Hz. This is rather useful negative information which has been reported to various levels of accuracy at various frequency bands by other experimenters over the past 15 or 20 years. By analyzing long records of Doppler data, extending over several days, it might be possible to detect coherent sources of GW at a level below the plasma curve (2) in Fig. 1. However, it is doubtful that any coherent sources exist in the Doppler detection band with strain amplitudes much above 10^{-15} [9].

The probability of detecting GW by the Doppler technique can be increased substantially by simply replacing the current S-band uplink with an X-band link. The resulting noise environment is shown in Fig. 2. At the same time, improvements could be made in the DSN ground systems. We reflect this by a much improved, but reasonable, noise curve (1) for the receiver. The hydrogen maser curve (4) is based on the performance of selected "good" DSN masers now in hand. A comparison of Fig. 2 with Fig. 1 shows about an order of magnitude improvement with the addition of X-band uplink. The problem with

achieving the noise performance of Fig. 2 is that with the paucity of planetary missions planned for the 1980's there is presently no candidate spacecraft to carry an X-band transponder to the outer solar system.

If we look further ahead into the 1990's, it is possible that spacecraft radio systems will be flown with multifrequency capabilities. Plasma noise will not be a problem. Then, if the frequency standard is the limiting noise source, it will be important to use systems that are stable to 10^{-16} over 1000 sec. At this level, the Doppler system could be an important tool of observational astronomy in the VLF region of the GW spectrum. At this point, though, we would have to be concerned with tropospheric noise. Some improvement over line (3) in Fig. 2 could be achieved by atmospheric monitoring, but in the long run the best solution would be to remove the tracking station from the surface of the earth. Possibilities that come to mind are an orbiting space station or a permanent lunar base.

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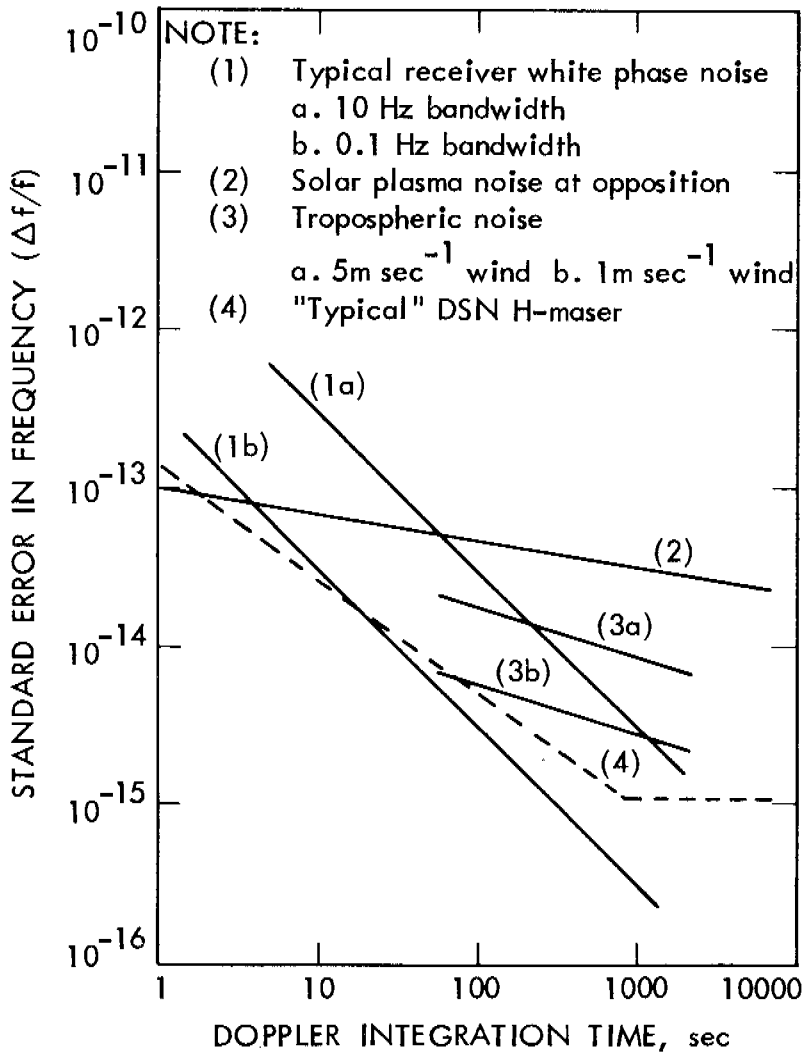

Fig. 1 Doppler Noise for Current S Band Systems

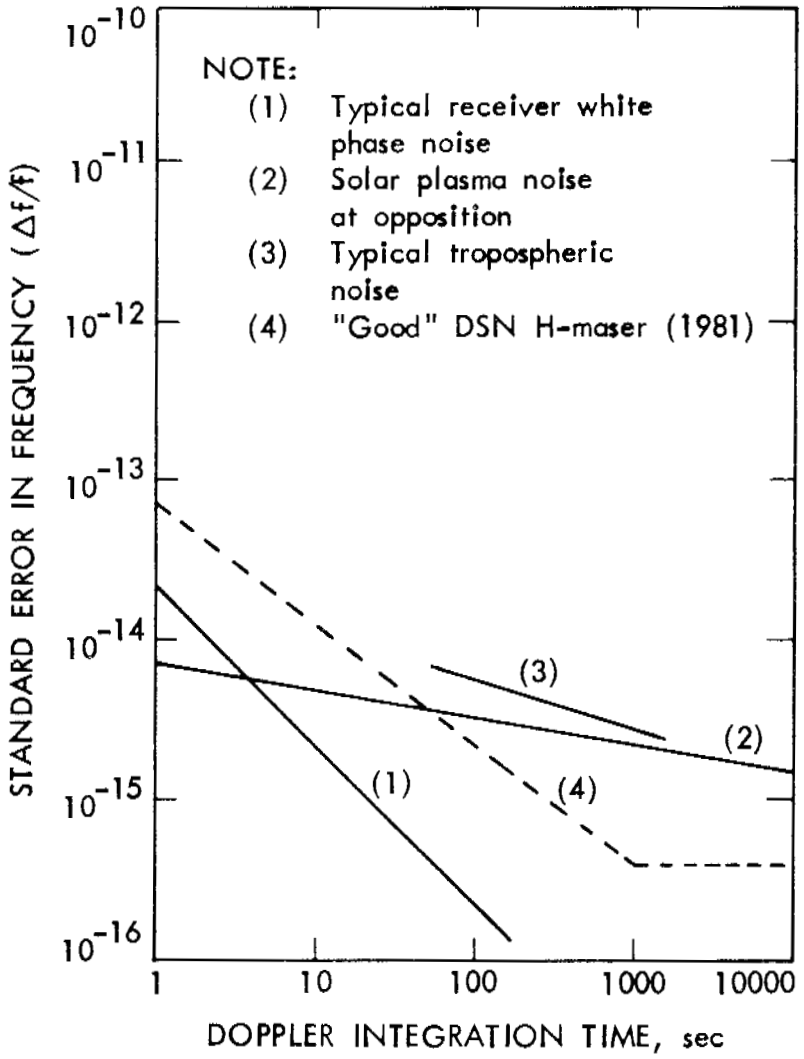

Fig. 2 Doppler Noise for Projected X-Band Systems

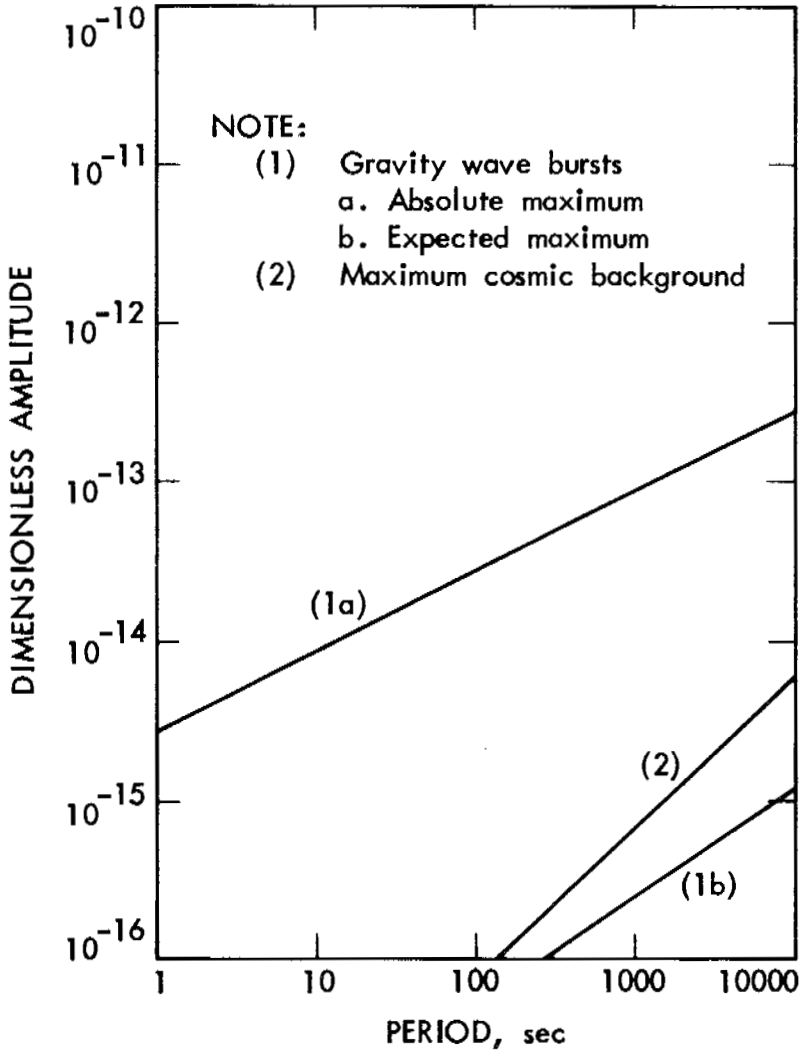

Fig. 3 Estimates of the Amplitude of Various GW Sources (After Thorne [8])