CRYOGENIC MASERS

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ABSTRACT

The long-term frequency stability of a hydrogen maser is limited by the mechanical stability of the cavity, and the magnitudes of the wall relaxation, spin exchange, and recombination rates which affect the Q of the line. Recent magnetic resonance studies of hydrogen atoms at temperatures below 1 K and in containers coated with liquid helium films have demonstrated that cryogenic masers may allow substantial improvements in all of these parameters. In particular the thermal expansion coefficients of most materials are negligible at 1 K. Spin exchange broadening is three orders of magnitude smaller at 1 K than at room temperature, and the recombination and wall relaxation rates are negligible at 0.52 K where the frequency shift due to the ^{4}He-coated walls of the container has a broad minimum as a function of temperature. Other advantages of the helium-cooled maser result from the high purity, homogeneity, and resilence of the helium-film-coated walls and the natural compatibility of the apparatus with helium-cooled amplifiers, which are necessary to take advantage of the intrinsically low thermal noise of the cooled cavity.

I. INTRODUCTION

The results on which this paper is based grew out of a program in which magnetic hyperfine resonance was used to study the behavior of hydrogen atoms at liquid helium temperatures (T < 4.2 K). The primary motivation for this program was (and still is) the possibility of producing high densities of atomic hydrogen gas at sufficiently low temperatures that quantum degeneracy effects, similar to those which occur in superfluid ^{4}He, might be observed. Since this paper is about masers, and not about superfluids, the latter subject will not be pursued any further. However the interested reader is referred to reviews by Berlinsky(1) and Hardy,(2) which discuss the problem of observing superfluidity in hydrogen gas and contain further details of our magnetic resonance work.

A natural dividend, accruing from our pursuit of superfluid hydrogen, has been the development of a practical working knowledge of the behavior of hydrogen atoms in a cryogenic environment. The ability, which we have developed, to maintain high densities of hydrogen atoms for long times and with extremely sharp magnetic resonance lines, is likely to lead to significant improvements in the stability of the hydrogen maser.

The idea of improving the frequency stability of hydrogen masers by lowering the maser temperature has been discussed by a number of authors. In particular Vessot et al.[3] noted that a thousand-fold improvement in the quality factor q (defined below) results from the fact that the spin exchange cross-section σ drops precipitously at low temperatures as does the thermal velocity $\bar{v}$. The main impediment to exploiting this advantage has been the lack of a suitable container for cold hydrogen-atoms. This problem has its analogue in room temperature masers where the development of fluorocarbon wall coatings was a necessary precursor to the successful operation of the hydrogen maser.

When a conventional maser, with Teflon-coated walls, is cooled to low temperatures, it stops operating somewhere below liquid nitrogen temperature (77 K) when the atoms begin to spend an excessive amount of time stuck to the Teflon, which both shifts their frequency and broadens the maser line. The natural solution is to use some less binding surface such as solid neon or, at lower temperatures still, solid H_2. However, at this point, the options quickly run out. The least binding surface, solid H_2, has a binding energy E_B for H of roughly 35 K.[4] This means that at very low temperatures ($T < 5$ K) essentially all the hydrogen atoms are on the wall, and the magnetic resonance lifetime is too short for maser operation. An important breakthrough in our work was the discovery that liquid ^{4}He-coated surfaces have a very low binding energy for hydrogen, $E_B = 1.15$ K.[5] Liquid ^{3}He is even better, with $E_B = 0.40$ K.[6] However ^{3}He is technically more difficult to work with, and hence we will concentrate in this paper on the use of liquid ^{4}He-coated walls. (See, however, Hardy and Morrow[7] for more details on the use of ^{3}He wall-coatings for hydrogen masers.)

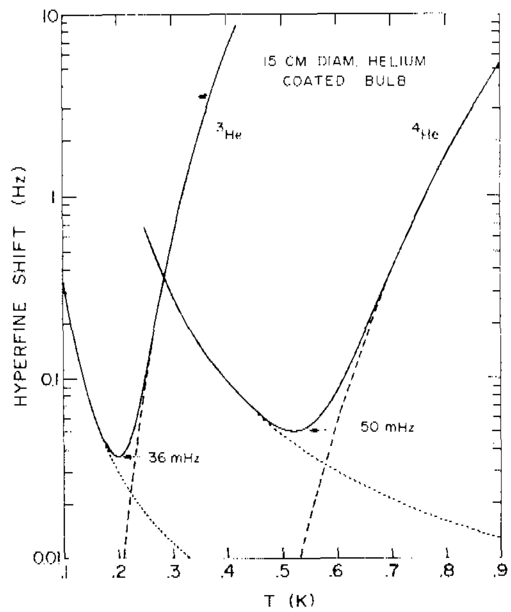

Fig. 1 – Predicted hyperfine shift vs temperature for a 15-cm-diameter bulb coated with liquid ^{3}He and ^{4}He.

Except at the lowest temperatures, $T \ll 1$ K, the vapor pressure of ^{4}He is substantial. This leads to a pressure shift of the hydrogen atomic resonance frequency, which is strongly temperature dependent, since it follows the vapor pressure of liquid ^{4}He. To minimize this effect one would like to work at a temperature where the pressure shift is negligible, say at $T \leq 0.4$ K. However this is just the region where the atoms start to stick to the ^{4}He surface, which also causes a shift of the same sign. Thus there is a cross-over temperature where the total, pressure shift plus wall shift, has its minimum, as shown in Fig. 1. For the nominal maser dimensions that we assume in this paper (see below) the minimum occurs at $T = 0.52$ K, which is thus the natural and, in fact, the only practical operating point for a ^{4}He-cooled hydrogen maser.

The remainder of this paper is organized as follows: Section II contains a description of the various sources of frequency instability in a hydrogen maser, along with estimates of the size of these effects in conventional masers. In Sec. III, the various factors affecting the frequency stability, such as line Q and power, are related to maser design parameters, such as incident beam flux and storage time. In Sec. IV, we consider specific design parameters for a maser operating at $T = 0.52$ K, and in Sec. V we estimate the stability that might be achieved in such a maser. We conclude that $\Delta f/f$ of order 2×10^{-18} should be obtainable, which is about 300 times better than the stability of conventional hydrogen masers.

II. SOURCES OF FREQUENCY INSTABILITY IN HYDROGEN MASERS

The main sources of frequency instability in a hydrogen maser are the following:

1) Thermal noise in the oscillator,
2) Receiver noise,
3) Time dependent frequency pulling due to fluctuations in the cavity frequency.

We will consider each of these in turn.

Thermal noise in the oscillator leads to random perturbations of the frequency of magnitude[8]

$$\left(\frac{\Delta f}{f}\right)_{osc} = \frac{1}{Q_\ell}\left(\frac{kT}{2P\tau}\right)^{1/2} , \qquad (1)$$

where $Q_\ell = \pi f_0 T_2$ is the Q of the maser line, $f_0 = 1,420,405,751.773$ is the frequency of the maser,[9] P is the power radiated by the atoms, and τ is the time of the measurement. In the situation in which the maser bulb, cavity, and isolator are all at the same physical temperature, T is that temperature. If the isolator is at a higher temperature, then the situation becomes more complicated.[7] Here, we assume that all components are at the same temperature.

The most important contribution to receiver noise arises in the first stage amplifier and contributes[3]

$$\left(\frac{\Delta f}{f}\right)_{rec} = \frac{1}{\omega_0 \tau}\left[\frac{Bk(T+T_N)}{P_A}\right]^{1/2} \qquad (2)$$

where B is the effective noise bandwidth, T_N is the noise temperature of the amplifier, and P_A is the power delivered to the amplifier. The relation of P_A to P is determined by the loaded Q of the cavity and the coupling Q, Q_c

$$P_A = \frac{Q}{Q_c} P \qquad (3)$$

and

$$\frac{1}{Q} = \frac{1}{Q_0} + \frac{1}{Q_c} \quad , \qquad (4)$$

where Q_0 is the unloaded cavity Q. In general, conventional masers are lightly coupled while, as we shall see, it will usually be best to overcouple a helium-temperature maser.

Fluctuations in the cavity frequency, δf_c, pull the maser frequency by an amount

$$\Delta f = \frac{Q}{Q_\ell} \delta f_c \quad . \qquad (5)$$

The source or sources of δf_c and their spectral distribution are not well understood. In conventional masers δf_c is often attributed to temperature fluctuations since the cavity frequency is quite measurably temperature dependent. However, mechanical instabilities almost certainly contribute also. The shift of the maser frequency Δf is proportional to the loaded Q. It may be minimized by overcoupling, which is only practical at low temperatures.

A measure of $\Delta f/f$ is obtained by heterodyning two masers to a low difference frequency, say 1 Hz, and then measuring the time required to count a prescribed number of cycles of the difference frequency, say 10^n, $n = 0, 1, 2 \ldots$ The difference of two successive times allows calculation of the two-sample or Allan variance $\sigma(2, T, \tau, B)$ where T (here only) is the time between the beginning of one measurement and the beginning of the next, τ is the actual measurement time, excluding "dead time," and B is the bandwidth of the receiver. If the various contributions to $\Delta f/f$ are independent then $\sigma(2, T, \tau, B)$ is equal to the rms sum of these contributions. Measurements[3] of $\sigma(2, T, \tau, B)$ for two VLG-11 masers are shown in Fig. 2. Both the short term τ^{-1} behavior of Eq. 2 and the long term $\tau^{-1/2}$ behavior of Eq. (1) are evident for $\tau < 10^3$ s. For still longer times, σ begins to increase approximately as $\tau^{1/2}$, as the result of a systematic drift in relative frequency of the two masers.

This drift is caused by pulling due to changes in the cavity frequencies and the effect is smaller than 10^{-15} at $\tau = 10^3$ s. The minimum value of $\Delta f/f$ achieved in Fig. 2 is about 6×10^{-16} for $\tau \approx 1$ h.

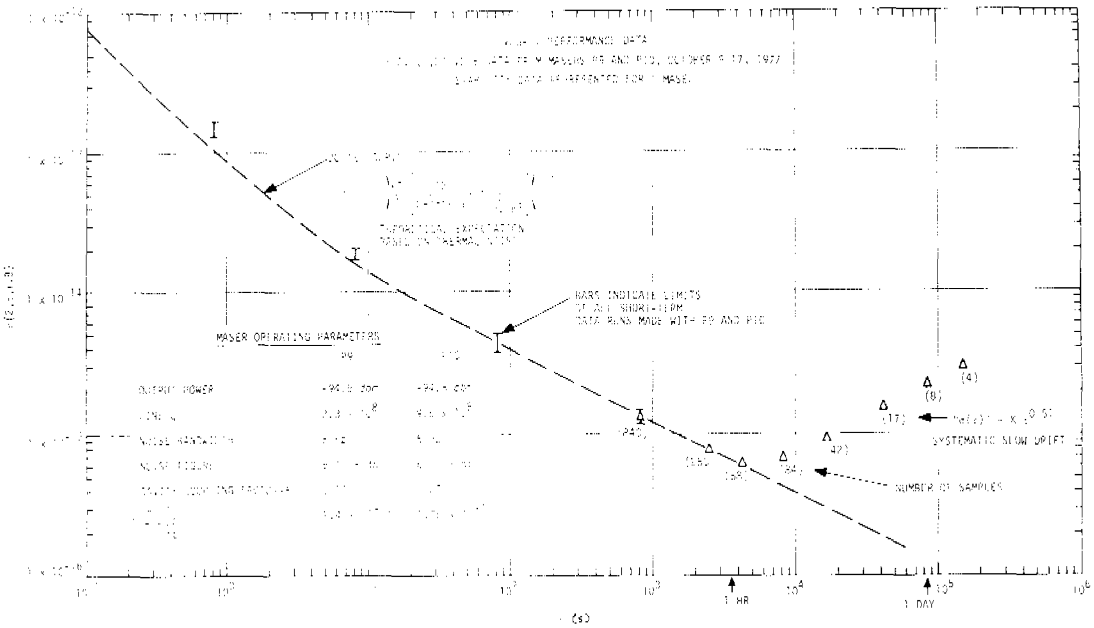

Fig. 2 – VLG-11 stability data. $\sigma(2, \tau, \tau, B)$ vs τ for masers P.9 and P.10, October 9-17, 1977.

III. RELATION OF FACTORS DETERMINING FREQUENCY STABILITY TO THE OPERATING PARAMETERS OF A MASER

The condition for maser operation is simply that the power emitted by the atoms equal the power absorbed by or coupled out of the cavity. If the cavity is tuned to the maser frequency $\omega = 2\pi f_0$, then the power may be written as[10]

$$P = \frac{1}{2}\,\hbar\omega\,(I - I_0\,T_b^2/T_1T_2) \quad , \tag{6}$$

where I is the net incident flux (i.e., the difference in flux of atoms in the $F = 1$, $m_F = 0$ and $F = 0$, $m_F = 0$ states, in atoms per second). T_b is the average time that an atom spends in the bulb, and T_1 and T_2 are, respectively, longitudinal and transverse relaxation times. T_1 measures the rate with which the level populations achieve thermal equilibrium, and T_2 is the time required for a collection of radiating atoms to lose phase coherence. The flux I_0 is defined as

$$I_0 = \hbar V_c/(4\pi\mu_B^2\eta\,QT_b^2) \quad , \tag{7}$$

where V_c is the volume of the cavity, μ_B is the Bohr magneton, and η is the "filling factor" defined as

$$\eta = \langle H_z\rangle^2_{bulb}/\langle H^2\rangle_{cavity} \quad . \tag{8}$$

Contributions to the relaxation rates, T_1^{-1} and T_2^{-1}, result from atoms leaving the bulb, from interactions with the wall and from spin exchange interactions between H atoms. Explicitly one has

$$\frac{1}{T_1} = \frac{1}{T_b} + \frac{1}{T_1^w} + \frac{1}{T_1^{se}} \tag{9a}$$

$$\frac{1}{T_2} = \frac{1}{T_b} + \frac{1}{T_2^w} + \frac{1}{T_2^{se}} \quad , \tag{9b}$$

where

$$\frac{1}{T_1^{se}} = \frac{2}{T_2^{se}} = \sigma \bar{v} n_H \quad . \tag{10}$$

Here σ is the spin-exchange cross section ($\sigma = 2.31 \times 10^{-15}$ cm^2 at room temperature)(11) and $\bar{v}$ is the thermally averaged relative velocity of pairs of hydrogen atoms ($\bar{v} \approx 2 \times 10^4 \sqrt{T}$ cm/sK$^{1/2}$). Thus $\sigma\bar{v} \approx 10^{-9}$ cm^3/s at room temperature. The density of hydrogen atoms in the bulb n_H may expressed in terms of the total flux $I_{tot} \gtrsim 2$ I, if magnetic hexapole state selection is employed, the volume of the bulb, and T_b,

$$n_H = I_{tot} T_b / V_b \quad . \tag{11}$$

For liquid ^{4}He, $T_2^w > 500$ s at T = 0.52 K with A/V = 0.4 cm^{-1} and $T_1^w \gg T_2^w$. Neglecting $(T_{1,2}^w)^{-1}$ results in a great simplification in the analysis of Eq. (6). For the low temperature maser at low flux, spin exchange is unimportant, and T_1 and T_2 are essentially equal to T_b. Then Eq. (6) says that the maser turns on for $I > I_0$, the threshold flux. As I is increased, n_H increases and spin exchange causes T_1 and T_2 to become short. Eventually, at high flux the second term in Eq. (6) again dominates and the maser turns off. Somewhere in between, at I_{max}, the maser power passes through a maximum. The relevant scale of flux at which spin exchange becomes important can be inferred by examining the quantity

$$\frac{T_b}{T_2^{se}} = (I_{tot}/I) \frac{\sigma\bar{v}\, I\, T_b^2}{2V_b} \equiv (I/I_1) \quad . \tag{12}$$

The quantity $I_{tot}/I \approx 2$ is innocuous. It differs from 2 only to the extent that state selection is imperfect or that there are Majorana transitions within the F = 1 manifold. The quantity

$$I_1 = \frac{2V_b}{\sigma\bar{v}\, T_b^2} (I/I_{tot}) \tag{13}$$

is the flux at which the spin exchange width $(T_2^{se})^{-1}$ equals the rate at which atoms leave the bulb, $1/T_b$. The ratio of the fluxes

$$q \equiv I_0/I_1 \tag{14}$$

is the same as the quality factor defined by Kleppner et al.[8] when $1/T_1^W$ and $1/T_2^W$ are negligible. Small values of q, which result when the spin exchange rate constant $\sigma\bar{v}$ is small, correspond to a "good" quality factor. Kleppner et al.[8] showed that the maximum value of q for which a maser can operate is q = 0.172.

In terms of q and the quantity I/I_1 the maser power is

$$P = \frac{1}{2}\hbar\omega I_1 \left[-q + (1-3q)(I/I_1) - 2q(I/I_1)^2\right] \quad . \tag{15}$$

This function is shown in Fig. 3a for $0 \le q \le 0.16$ ($q = 0.1$ is typical for conventional masers). In Fig. 3b Eq. (15) is plotted for q = 0.024 and q = 0.0024, which are appropriate for low temperature. From Eq. (15) we find that for $q \ll 1$ the maser threshold is $I = qI_1 = I_0$. The maximum in P occurs at $I_{max} = I_1/4q$, and the maser turns off at $I = I_1/2q$. For small I

$$P \approx \frac{1}{2}\hbar\omega I_1 \left[(I/I_1) - q\right] \quad . \tag{16}$$

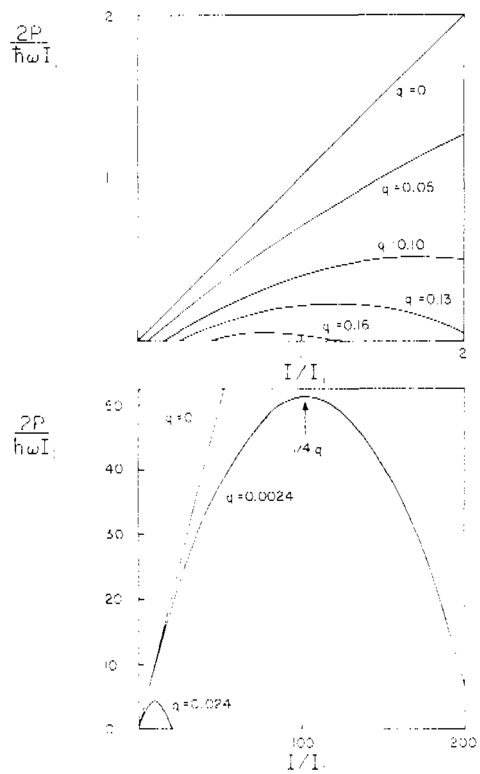

Fig. 3 – Power in units of $\hbar\omega I_1/2$ versus incident flux in units of I_1 as defined in Eq. (13) for various values of q, defined by Eqs. (7), (13) and (14). (a) $0 \le q \le 0.16$ and (b) q = 0.024 and 0.0024.

For $q \ll 1$ as is the case at low T, the long-term noise, Eq. (1), is minimized by operating at $I = I_1$. To see this define $x = I/I_1$. Then Eq. (1) may be written as

$$\left(\frac{\Delta f}{f}\right)_{osc} = \frac{2}{\omega T_b} \left(\frac{kT}{\hbar\omega I_1 \tau}\right)^{1/2} \frac{1+x}{\sqrt{x-q}} \quad , \tag{17}$$

where we have used the fact the $1/T_2 = (1+x)/T_b$ (cf. Eq. (12)).
Equation (17) is optimized by $x = 1+2q \approx 1$ since $q \ll 1$ by hypothesis. It is also noteworthy that, from Eq. (13) since $I_1 \sim T_b^{-2}$, Eq. (17) is independent of T_b. For $I = I_1$ and $q \ll 1$ one obtains

$$\left(\frac{\Delta f}{f}\right)_{osc} = \left(\frac{8\,kT\sigma\bar{v}}{\hbar\omega^3 V_b \tau} \frac{I_{Tot}}{I}\right)^{1/2} \quad . \tag{18}$$

Under the same assumptions the short-term receiver noise is given by

$$\left(\frac{\Delta f}{f}\right)_{rec} = \left[\frac{k(T_N+T)\,B\sigma\bar{v}}{\hbar\omega^3\, V_b} \frac{Q_c}{Q} \frac{I_{Tot}}{I}\right]^{1/2} \frac{T_b}{\tau} \quad , \tag{19}$$

which is proportional to T_b. Hence adjusting T_b affects the ratio of long-term noise to short-term noise for a given value of the averaging time τ.

Other factors that can affect the stability of the maser are the spin exchange shift $\Delta\nu_{ex}$, which depends on the incident flux, and cavity pulling, if the cavity is mistuned, which also depends on the incident flux through the line Q_ℓ (see Eq. (5)). Fortunately these two effects can be arranged to offset seach other. Crampton[12] has shown that the sum of these two effects may be written as

$$\Delta\nu = \left[\frac{2Q}{\omega}(f_c - f_0) - \frac{1}{4\pi}\, q S\, \frac{I}{I_{tot}}\right] \frac{1}{T_2} \quad . \tag{20}$$

The factor I/I_{tot} just cancels an identical factor in q, and S is the ratio of thermally averaged spin exchange shift to width cross section ($S = \lambda^+/\sigma$). Equation (20) shows that if the cavity is properly mistuned then fluctuations in $1/T_2$ will not affect the frequency. Of course the effect of fluctuations in f_c itself cannot be avoided by this technique.

IV. OPERATING PARAMETERS FOR T = 0.52 K

By now it is clear that the crucial difference between conventional masers and a helium temperature maser is the drastic reduction of the spin exchange broadening parameter $\sigma\bar{v}$ at low T. This is illustrated in Fig. 4 where $\sigma\bar{v}$ and the shift parameter $\lambda^+\bar{v}$ are shown plotted versus T. $\sigma\bar{v}$ is very close to 10^{-12} cm^3/s at 0.52 K, which is 10^{-3} of its room temperature value. The curves in Fig. 4 are theoretical calculations by Allison[13] for T > 10 K and by Berlinsky and Shizgal[14] for T < 10 K. Experimental points[15-17] both at high temperatures and at 1.2 K confirm the theoretical predictions. The shift parameter changes sign twice in going from room temperature to T < 1 K. At T = 0.5 K the ratio S of shift to width has the value 95 compared to room temperature where $S = 0.2$. However it is the product qS that appears in the tuning (Eq. (20)) and this product is smaller at 0.5 K than at 300 K for values of the loaded Q larger than about half the Q of the room temperature maser.

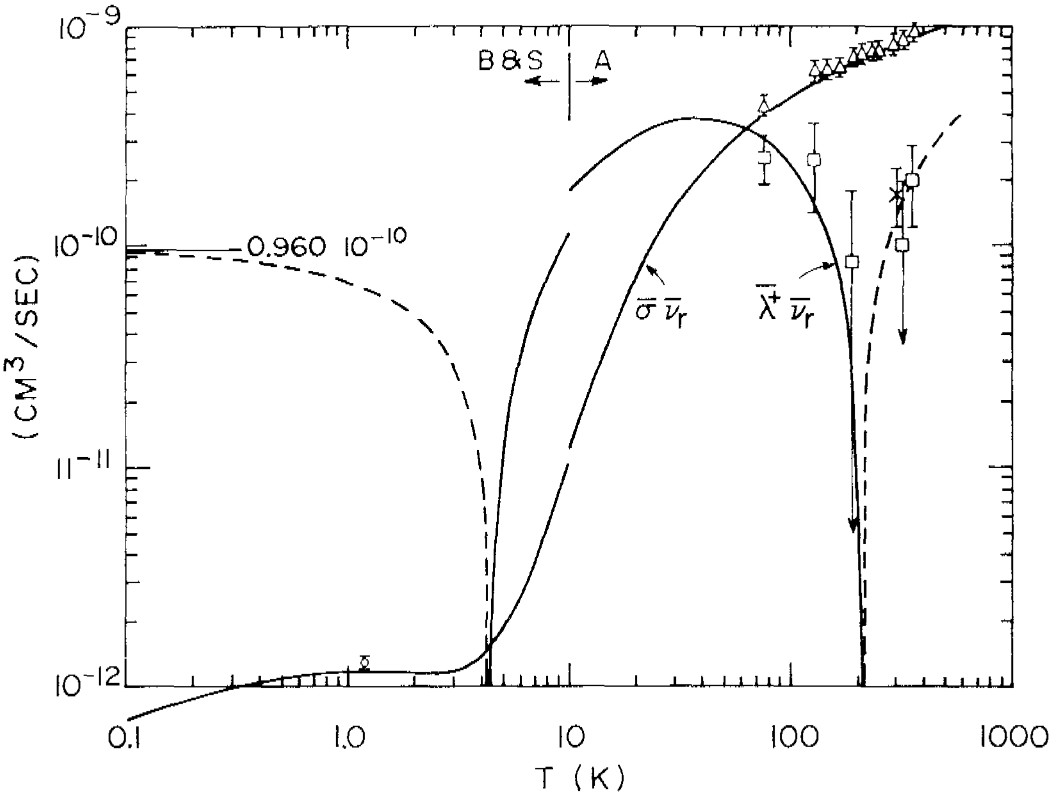

Fig. 4 – Spin exchange width (σ) and frequency shift (λ^+) cross sections times relative velocity. Dashed lines represent negative values of $\lambda^+ \bar{v}$. B+S refers to the theory of Berlinsky and Shizgal, Ref. 14. Symbols are experimental data: $\Delta = \sigma\bar{v}$ and $\square = \lambda^+\bar{v}$ from Ref. 15. x = $\lambda^+\bar{v}$ from Ref. 16. o = $\sigma\bar{v}$ from Ref. 17.

At this point it is useful to choose nominal values for the maser parameters so that estimates can be made of the operating conditions and potential stability of the low temperature maser. For convenience we choose parameters appropriate to conventional masers: $V_b = 1.8 \times 10^3$ cm^3, $V_c = 1.3 \times 10^4$ cm^3, $\eta = 3$. Our analysis will be quite insensitive to the value of the loaded Q although we assume $Q \ll Q_0$, the intrinsic Q of the cavity. We expect $Q_0 \sim 10^5$ and $10^2 < Q < 5 \times 10^3$. The minimum possible value of Q is that which implies $q = 0.172$. From Eqs. (7), (13), and (14), taking $I_{tot}/I = 2$ this value is $Q_{min} = 14$ at T = 0.52 K. For Q = 100, q = 0.024, and the maximum in the power curve occurs at $I \gg I_1$. (See Fig. 3b.) For Q = 1000 the condition $q = 0.0024 \ll 1$ is well satisfied. The only advantage of the smaller value, Q = 100, is that the effect of cavity pulling is further suppressed. However we shall see in the next section that this effect is likely to be negligible even for $Q = 1000$. We would like to comment at this point that the final choice of a value of Q may require consideration of additional contributions to δf_c arising from overcoupling. In fact it is the product $Q\delta f_c$, which should be optimized by a proper choice of Q.

The value $Q = 1000$ implies a threshold flux $I_0 = 4.2 \times 10^{12}\ \mathrm{s}/T_b^2$. Independent of Q we have $I_1 = 1.8 \times 10^{15}\ \mathrm{s}/T_b^2$. Since $I_{Tot} \approx 2\ I$, operating at $I = I_1$ requires a total flux of $3.6 \times 10^{15}\ \mathrm{s}/T_b^2$. For $T_b = 1$ s, I_{Tot} is about 100 times larger than that of a conventional maser where the limitation results from the speed of the vacuum pumps. At low temperatures pumping is not such a problem because helium temperature surfaces cryopump molecular hydrogen with great efficiency. It may also be desirable to work with a larger value of T_b if the short term noise is not too severe. Taking $T_b = 10$ s reduces I_1 to quite a manageable rate.

A number of other advantages appear when operating at liquid helium temperature, which are peripheral to the maser bulb and cavity. For example, one has available liquid helium-cooled pre-amplifiers, based on GaAs FET's, with noise temperature T_N of 10 K, which greatly reduce receiver noise. Magnetic state selection is also more efficient at low T because atoms emerging from low temperature sources move more slowly and hence are easier to focus. On the other hand a variety of complications result from the presence of the liquid ^{4}He film. For instance the vapor pressure of ^{4}He at this temperature is slightly higher than one would like. The mean free path for hydrogen in the ^{4}He vapor(7) will be about 1.4 cm at $T = 0.52$ K, which will make it difficult to inject the hydrogen atoms in a beam. Also the tendency of the ^{4}He film to flow toward warmer temperature regions where it evaporates and is then pumped back down into the bulb implies a region of refluxing ^{4}He vapor near the entance of the bulb. In certain circumstances this can act as a ^{4}He vapor diffusion pump(18) for the hydrogen atoms, and this effect might possibly prove useful in the design of the apparatus.

V. ESTIMATES OF THE POTENTIAL STABILITY OF A HYDROGEN MASER OPERATING AT T = 0.52 K

We are now in a position to estimate the actual sizes of the various contributions to the frequency fluctuations of a maser operating at 0.52 K. The physical parameters of the maser that we consider are those mentioned above: $V_b = 1.8 \times 10^3\ \mathrm{cm}^3$, $V_c = 1.3 \times 10^4\ \mathrm{cm}^3$, $\eta = 3$. The values of T_b and Q will be left free to optimize the performance of the maser. Then for $I_{Tot}/I = 2$, $I = I_1$, and $\sigma\bar{v} = 10^{-12}\ \mathrm{cm}^3/\mathrm{s}$, Eq. (18) has the value

$$\left(\frac{\Delta f}{f}\right)_{osc} = \frac{2.9 \times 10^{-17}}{\tau^{1/2}}\ \mathrm{s}^{1/2} \quad , \tag{21}$$

and for $T_N = 10$ K and $B = 6$ Hz, Eq. (19) gives

$$\left(\frac{\Delta f}{f}\right)_{rec} = 1.2 \times 10^{-16}\ T_b/\tau \quad . \tag{22}$$

T_b is then determined by equating the short and long term noise at a suitable averaging time such as $\tau = 10^3$ s. The result is $T_b = 8$ s, and then the rms sum of the two contribution is $\sqrt{2}$ times the long term noise, i.e., $\Delta f/f = 1.3 \times 10^{-18}$ for $\tau = 10^3$ s and $T_b = 8$ s. The effect of choosing T_b is to determine the operating flux I_1 through Eq. (13). $T_b = 8$ s implies $I_1 = 2.8 \times 10^{13}$ s^{-1} and hence $I_{tot} \approx 5.6 \times 10^{13}$ s^{-1}, which is comparable to fluxes employed in conventional masers.

This value of I_{tot} determines n_H, the density of atoms in the bulb, to be $n_H = 2.5 \times 10^{11}$ cm^{-3}. We have measured the rate constant for recombination for H into H_2 at this temperature in the presence of liquid ^{4}He. The rate constant which we measure implies a recombination lifetime $\tau_b = 1.4 \times 10^5$ s at this density of hydrogen. Thus for $T_b = 8$ s nearly all the atoms leave the bulb before they recombine.

Next we consider the effect of time dependent pulling due to fluctuations in the cavity frequency. At room temperature these fluctuations are thought to result from the nonzero temperature dependence of the frequency of the cavity. Such effects will be totally negligible below 1 K where thermal expansion coefficients of most technical materials are extremely small. For example the thermal expansion coefficient of copper at T = 0.5 K is less than 10^{-5} of its room temperature value. Mechanical instabilities are more difficult to analyze. However it is unlikely that they will be more severe at low T.

In any case one can minimize pulling effects by reducing the value of Q/Q_ℓ in Eq. (5). At room temperature this quantity is typically around 2×10^{-5}. For the low temperature maser, $Q_\ell = \pi f_0 T_2 = 1.8 \times 10^{10}$ for our optimal case ($T_2 = T_b/2 = 4$ s). Then, taking Q = 1000, $Q/Q_\ell = 5.6 \times 10^{-8}$, and cavity pulling should be down by a factor of 3×10^{-3}. Since cavity pulling effects are already well below the 10^{-15} level for $\tau = 10^3$ s at room temperature, this lower value of Q/Q_ℓ should allow operation with stabilities $\Delta f/f$ of order 10^{-18}.

An obvious additional source of frequency instability in the low temperature maser is the temperature dependence of the wall shift. Near the minimum frequency f_{min} the temperature dependence may be represented as

$$f(T) = f_{min} - 3.73\,(T - T_{min})^2 \quad . \tag{23}$$

If the fractional frequency stability required is $\Delta f/f$ then the temperature must be maintained within ΔT of T_{min} where

$$\Delta T = \left(\frac{\Delta f/f}{3.73/f}\right)^{1/2} \quad . \tag{24}$$

Taking $\Delta f/f = 10^{-18}$ implies $\Delta T = 20\,\mu$K. This level of temperature stability is well within the capability of modern low temperature technology. To illustrate this point we show in Fig. 5 recent high resolution measurements, by J.A. Lipa of Stanford University, of the heat capacity of liquid ^{4}He near the λ transition at 2.2 K.(19) Controlled temperature drift rates in this experiment ranged from 1 K/s to 9×10^{-8} K/s and the temperature resolution was 6×10^{-10} K for a bandwidth of 1 Hz at 2.2 K. Note that the entire width in temperature covered by Fig. 5a is the width within which a low temperature maser would have to be controlled.

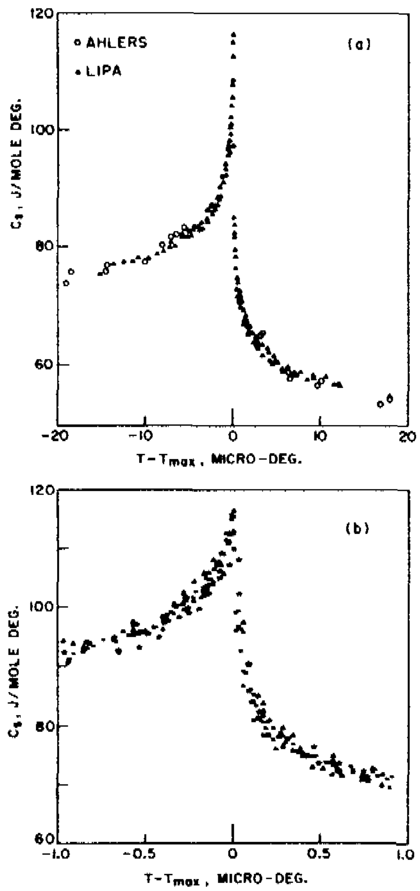

Fig. 5 – Heat capacity measurements close to T_λ for liquid ^4He from Ref. 19. This figure illustrates the state of the art of temperature control at 2.2 K, which is not very different from the situation at 0.52 K.

We conclude that the achievement of frequency stability of order 2×10^{-18} for a measuring time $\tau = 10^3$ s is a plausible objective for helium-cooled hydrogen masers operating at the temperature where the wall shift has its minimum. This will not be an easy goal to achieve, and one can say with confidence that many technical difficulties will arise in the course of development, which will have to be dealt with. However it is also true that we have learned quite a lot in the past few years about hydrogen atoms in the presence of liquid-helium-coated walls, and that this knowledge has allowed us to construct a fairly complete scenario of how a cold maser might operate. Based on this scenario the stability estimates given above appear reasonable.

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QUESTIONS AND ANSWERS

DR. VESSOT:

There have to be questions. The reason that there have to be questions is we are now, I think, on the threshold of something that's very, very good. And I think the questions are more likely in the line of speculations to what can you do with two in 10 to the 18.

I can think of gravitational waves. But then that would be stealing other people's thunder, and that's another session this afternoon.

DR. REINHARDT:

In trying to achieve a maser at these low temperatures I think the real glitch is going to be state selection, because the interaction with the high temperature state selector and the low temperature environment.

However, there is an alternative which you can just use the natural occurring population difference and use an absorbing cell. That has many problems.

Do you want to comment on that?

DR. BERLINSKY:

Well, I guess if we made a frequency standard which was just looking at the magnetic resonance line, then the sense corresponds to what you say.

Let me comment in this way. The philosophy I believe that you have to use to design a low temperature maser is just to think of everything in terms of low temperatures.

Now I agree with you that it may be difficult to make a room temperature state selector which is compatible with a low temperature maser. But I would never do it that way. I would make a low temperature state selector, just as we have a low temperature amplifier, and somebody is going to have to build a low temperature isolator.

And I think that you can design a state selector for us in this environment, which would probably work extremely well. And, in fact, I'd love to talk about how to do that.

DR. VESSOT:

Let's have another question. And then you can argue that one over lunch.

DR. BERLINSKY:

Okay.

DR. MICHEL TETU, Laval University

I want to underline the fact that you used, in order to get provision of the short-term and mid-term stability of your maser, an equation that might be not exactly the right one. And at the end of this session I will give a talk in which I will recall these relations.

And I suggest that you look at a paper by Claude Audoin at the Philadelphia meeting on the 33rd Symposium on Frequency Control. To get there a reevaluation of the theoretical expression for short-term stability of maser.

And I think people have to be aware of this calculation.

DR. VESSOT:

Is it better or is it worse? Give us the bottom line Michel.

DR. TETU:

I think that it will be a little worse, but not really more than an order of magnitude.

DR. BERLINSKY:

Can you just say what factor it is different?

DR. TETU:

You have already a factor of four for the thermal noise spectral density, and also the fact that the noise from the receiver is not expressed exactly as the f-factor. But you have to add it separately with an f for the noise figure, minus one, times the spectral density of the thermal noise.

DR. WINELAND:

Is there any particular reason here for doing your calculations in terms of oscillating masers?

DR. VESSOT:

He's been prejudiced by me. But carry on.

DR. BERLINSKY:

Yeah, there are several aspects of my calculation which are tied to what look to me to be conventional maser designs. And I think as we get further into the business that many things will change.

DR. WINELAND:

But there's no particular reason?

DR. BERLINSKY:

No. I would say no.

DR. WALLS:

Except for the problem of a local oscillator. We don't have any local oscillators which are 10 to minus 15, or 10 to minus 16, or 17 at a second that would allow to realize these incredible numbers you are talking about.

I think a self-excited oscillator is probably the only way to reach those levels.