ATMOSPHERIC LIMITATIONS TO CLOCK SYNCHRONIZATION AT MICROWAVE FREQUENCIES

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ABSTRACT

Clock synchronization schemes utilizing microwave signals that pass through the earth's atmosphere are ultimately limited by our ability to correct for the variable delay imposed by the atmosphere. The atmosphere is non-dispersive at microwave frequencies and imposes a delay of roughly 8 nanosec times the cosecant of the elevation angle. This delay is composed of two parts, the delay due to water vapor molecules (i.e. the "wet" delay), and the delay due to all other atmospheric constituents (i.e. the "dry" delay). Water vapor contributes approximately 5 to 10% of the total atmospheric delay but is highly variable, not well mixed, and difficult to estimate from surface air measurements. However, the techniques of passive remote sensing using microwave radiometry can be used to estimate the line of sight delay due to water vapor with potential accuracies of 10 to 20 picosec. The devices that are used are called water vapor radiometers and simply measure the power emitted by the water vapor molecule at the 22.2 GHz spectral line. An additional power measurement is usually included at 31.4 GHz in order to compensate for the effect of liquid water (e.g. clouds). The dry atmosphere is generally in something close to hydrostatic equilibrium and its delay contribution at zenith can be estimated quite well from a simple barometric measurement. At low elevation angles one must compensate for refractive bending and possible variations in the vertical refractivity profile. With care these effects can be estimated with accuracies on the order of 30 picosec down to elevation angles of 10 degree.

I. INTRODUCTION

During the past decade we have witnessed a steady improvement in our ability to synchronize clocks on a global basis. Techniques such as Very Long Baseline Interferometry (VLBI) or any of several schemes that utilize earth orbiting satellites such as the Global Positioning System (GPS) offer the prospect of sub-nanosecond clock and frequency comparison. Atmospheric errors have not been a major contributor to the error budget in these techniques but as we approach the nanosecond (ns) level of accuracy, as our instrumentation and experimental technique improves, the atmospheric delay effects begin to take on the aspect of a limiting error source (Resch,1980). This paper is intended to quantify the magnitude of these atmospheric effects at microwave frequencies and review the extent to which they can be reduced with technology that is currently available.

II. ATMOSPHERIC DELAY

At microwave frequencies it is a good approximation to consider the atmosphere to be non-dispersive. An elemental volume of air is characterized by its index of refraction n(s), so that the total delay experienced by a signal from an extraterrestrial source (neglecting bending) is;

$$\tau_{ATM} = \int_L n(l)/c dl \tag{1}$$

Where c is the vacuum speed of light and the integral is evaluated along the ray path L whose line element is *dl*. It is convenient to define a parameter N, called the refractivity, that is a measure of the departure of the index of refraction from unity.

$$N = (n-1)10^6 \tag{2}$$

We can now write the "extra" delay imposed by the atmosphere (i.e. over and above the geometric delay) as,

$$\Delta\tau = 10^{-6} \int_L N(l) dl/c \tag{3}$$

If we are trying to synchronize clocks by observing an extraterrestrial source then the entire problem of accounting for atmospheric effects reduces to estimating this simple integral.

Using the molecular properties of atmospheric constituents it is possible to derive an analytic expression for the refractivity (Bean and Dutton, 1968). A simple formulation for the refractivity has been given by Smith and Weintraub (1953) as,

$$N = 77.6(P/T + 4810e/T^2)$$

$$N = N_D + N_W \tag{4}$$

Where, T is the temperature in Kelvin (K), P the total pressure in millibar (mb), and e is the partial pressure of water vapor in mb. This expression is accurate to 0.5% over the range of temperature, pressure, and vapor content normally found in the atmosphere. Note that the refractivity can be decomposed into two components. One component we call the "wet" component because it depends primarily on the density of water vapor (i.e. a polar molecule), and the other we call the "dry" component in which we lump the effects of all atmospheric gases (including water vapor) but is dominated by the most abundant molecules of oxygen and nitrogen.

Hence, the atmospheric delay correction is simply decomposed into two separable problems. Using elementary definitions, the dry and wet atmospheric delay corrections can be written as;

$$\Delta\tau_D = \int_L \rho_D(l)dl/c \tag{5}$$

$$\Delta\tau_W = \int_L \rho_v(l)/T dl/c \tag{6}$$

where ρ_D is the density of dry air, ρ_v is the vapor density, and T is the temperature. Estimating the dry delay is equivalent to evaluating the integral of the dry air density along the ray path. Estimating the wet delay is equivalent to evaluating the integral that contains the vapor density divided by the temperature, again along the entire ray path.

III. ZENITH DELAY VALUES AND MAPPING FUNCTIONS

At sea level under average conditions the total atmospheric delay at the zenith is approximately 8 ns. The dry atmospheric delay at the zenith is just a bit less than 8 ns and is dominated by the gaseous form of oxygen and nitrogen. These components are well mixed throughout the atmosphere and hydrostatic equilibrium is a reasonable approximation. The wet delay is highly variable and can range from practically zero up to 1 ns at the zenith. Although the wet delay contributes less than 10% of the total atmospheric delay it dominates the variablity and will take 99% of your effort should you require its accurate calibration.

The reason that the wet delay is such a problem lies in the fact that water is not a well mixed atmospheric constituent, it occurs in all three phases (solid, liquid, and gas). The mixing ratio is driven primarily by thermal processes in the lower atmosphere which means that it is difficult to estimate the wet zenith delay using only surface meteorological measurements. Nevertheless, one can model the water vapor and estimate a zenith delay. The problem with a water vapor model is the accuracy of the resulting estimate which must be judged in the context of the goals for a particular experiment. Depending on how a set of observations is constructed, it may be possible to solve for the zenith values of atmospheric delay with higher confidence than is afforded by a model.

If you have ever tried to synchronize clocks with VLBI or by using satellites you will have noticed that the sources are never at the zenith. Zenith values of the delay correction must be mapped to the line of sight to the radio source. If we assume that the atmosphere is homogeneous and plane-layered, then the delay along an arbitrary line of sight (LOS) is simply,

$$\Delta\tau_{LOS} = \Delta\tau_z \csc E \tag{7}$$

where $\Delta\tau_z$ is the zenith delay and E is the elevation angle (azimuthal symmetry is implied in the assumption of homogenieity). This simple cosecant mapping function is generally quite adequate for elevation angles greater than 20 degrees. Of course the error in the zenith delay is also multiplied by the cosecant of the elevation angle, hence the premium on obtaining an accurate value of the zenith delay. For clocks that are separated by large distances it is not practical to restrict elevation angles to greater than 20 degrees.

If equation (5) is evaluated along a zenith ray path we see that it is simply the mass of air in a vertical column and can be measured with a barometer. If the total zenith delay at sea level is roughly 8 ns then an error of 1 mb in the barometric measurement corresponds to a delay error of 8 picosecond (ps). If we assume an elevation angle cutoff of 6 degrees, the line of sight atmospheric delay is approximately 80 ns (corresponding to 10 airmasses) and a 2 mb barometric measurement accuracy would map to 160 ps of line of sight delay error. Thus, with reasonable care of our barometer we can neglect measurement errors.

Much larger line of sight delay errors arise from three effects; 1) both the atmosphere and the ray path is curved, 2) errors in estimating the zenith vapor delay, and 3) the real atmosphere is not homogeneous.

If we use the simple mapping function we will make a 6 or 7 ns error at a 6 degree elevation because we did not account for earth curvature or ray bending. Variations in the real atmosphere and mis-modeling water vapor will account for another 1 or 2 ns error independent of the mapping function. Using a function only slightly more complicated than the cosecant we can take into account earth curvature and ray bending and reduce that portion of the error to less than 1 ns. There are long-term variations in the atmosphere (seasonal effects) that can be modeled, included with the mapping function and can remove perhaps 0.5 ns from the variable portion of the atmosphere. Finally, we are left with roughly 1 ns of variations that cannot be modeled but can be estimated using remote sensing to the 0.1 ns level down to 10 degree elevation.

There are at least a half-dozen mapping functions from various authors that account for atmospheric and ray path curvature at low elevation angles. In general they are semi-empirical formulas. In order to derive an improved mapping function one typically starts with some average profile of the refractivity, assumes horizontal homogeniety, performs ray-trace calculations at various elevation angles, and then notes that the delay as a function of elevation angle can be approximated by an analytic function containing a few parameters. Figure 1 compares some of the most popular mapping functions with actual ray trace calculations down to an elevation angle of 6 degrees. Shown are mapping functions from Lanyi (1984), Black (1978), Black and Eisner (1984), Chao (1974), Marini and Murray (1973), and Saastamoinen (1972).

The ray trace calculations that are used in Figure 1 as the "truth" are in fact based on the assumption of homogenieity. Bending of the ray path will depend upon the vertical density profile. Water vapor variations dominate the variations in the density profile and will exhibit variations on several timescales and may even exhibit horizontal gradients that are driven either by local topography or mesoscale weather patterns. If vertical soundings of temperature and relative humidity are available for a particular observing site then it is possible to identify the low frequency fluctuating components (e.g. seasonal variations) and incorporate them into the mapping function.

All of these mapping functions shown in Figure 1 offer significant improvement over the simple cosecant mapping. The most recent, by Gabor Lanyi at JPL has the distinct advantage of agreeing with ray trace calculations to better than 10 ps down to elevation angles of 6 degree. Lanyi's mapping function together with improved estimates of seasonal variability is now being tested on 7 years of VLBI data taken between the stations of the Deep Space Network. Preliminary indications are that this new mapping function exhibits one of the sought after qualities of accurate atmospheric delay correction - it improves the repeatability between experiments.

As mentioned earlier, the wet delay can also be modeled. Modeling is of course the least expensive method to account for atmospheric effects so there is a great deal of fiscal motivation to use them whenever possible and there is a plethora of models that can be used with varying degrees of statistical success to estimate the wet atmospheric delay. Berman (1976) has discussed several of these models. In general, one starts with the assumption that the vertical profile of vapor density is described by an analytic function, measure the surface value of vapor density, and use the model to estimate the zenith delay. The typical accuracy that is achievable is on the order of 100 ps at the zenith which translates to a 1 ns error at an elevation angle of 6 degree.

It is sometimes possible to structure an experiment so that it is possible to solve for the zenith delay. In this case, if one uses a good mapping function it is only the departures from homogenieity and temporal variations of the atmosphere that are error sources. If it is not possible to solve for the zenith delay and high accuracy is a requirement then one must directly estimate the line of sight vapor content. The technique that can be used falls in the category of passive remote sensing and is based on the fact that the water vapor molecule radiates weakly at the microwave frequency of 22.2 GHz. If the molecule is locked in the solid or liquid state the transition is inhibited so the spectral line is a direct indicator of water vapor. The technique has been reviewed by Hogg *et al.* (1983) and by Resch (1984) and will only be outlined here.

Figure 2 shows what an ideal radiometer would measure if it observed the zenith through a standard atmosphere between the frequencies of 10 to 300 GHz. The intensity or power level of the received radiation is shown along the vertical scale and is given in units of Kelvin which is a measure of the brightness temperature - the temperature that a black body would have if the black body were to replace the atmosphere and to deliver an equivalent amount of power to the radiometer. The lower curve shows the spectrum when there is no water vapor in the atmosphere and the upper curve is drawn for the case of a precipitable vapor of 2 gm/cm^2. You see several spectral feature between 10 and 300 GHz, one of which is the 22.2 GHz line from water vapor that was just mentioned. Under the assumption of low total absorption (i.e. less than 3 db) the strength of the line is proportional to the total amount of water vapor along the line of sight. In equation (6) we saw that the wet path delay can be cast into a form that very much resembles the integral of the vapor density along the line of sight. This means that we can use a radiometer operating at a frequency near 22.2 GHz to measure the intensity of radiation and develop an algorithm to then use the measurement in order to estimate the wet path delay. Unfortunately, nature does not let us off quite that easily.

Figure 3 shows the effects on the brightness temperature of liquid water assumed to exist as very small droplets similar to what exists in a cloud. This shows the brightness spectrum of the atmosphere for three cases: 1) no vapor and no liquid, 2) 2 gm/cm^2 of vapor and no liquid, and 3) 2 gm/cm^2 of vapor and 0.1 gm/cm^2 of precipitable liquid. This amount of liquid water has negligible effect on the delay but you can see that it has a very large effect on the measurement of the brightness temperature. We can either be content with a single channel radiometer that will operate only under clear sky conditions or we can add a second radiometer operating at a frequency off the water vapor line and use the second measurement along with the first to simultaneously estimate both the water vapor and liquid in the atmosphere. One can look at the second channel as the price you must pay in order to operate in both clear and cloudy conditions.

Instruments that are capable of estimating the line of sight delay have been described by Giraud *et al.* (1979) and by Resch *et al.* (1982). The absolute accuracy of the technique over the dynamic range that is experienced in the real atmosphere is addressed in Figure 4 (Resch, 1984) by comparing the amount of atmospheric water determined by two independent techniques. Along the vertical axis is plotted the wet delay that was inferred from an instrumented aircraft measurement. The aircraft carried a package of instruments that measured temperature, pressure, and relative humidity and flew predetermined flight paths that approximated various lines-of-sight through the atmosphere. The measurements were recorded and later converted to vapor density and integrated to obtain wet delay. The horizontal axis shows the vapor delay as determined by a water vapor radiometer (WVR) operated during the aircraft flight pointing along the flight path. The rms scatter of roughly 50 ps is the quadratic sum of the errors in both measurement techniques. If we rather generously assume that the errors in the aircraft measurement were on the order of 10% of the total delay then we can infer that the accuracy of the WVR is about 30 ps in the delay domain. Simulation calculations suggests that the theoretical limit of performance for the WVR is approximately 10 ps.

Figure 5, taken from Resch *et al.* illustrates the precision of two WVRs operating along with an interferometer in the Very Large Array located in New Mexico. The experiment was unusual in two respects. First, the baseline is only 7 km long and we would normally expect the atmosphere to be well correlated over that kind of separation however the data was taken during the summer when there was thunderstorm activity in the area and the atmosphere was very dynamic. Secondly, this is not a VLBI experiment, we were comparing the WVRs with a connected element interferometer whose phase stability is on the order of a few ps over a several hour period. The dotted line shows the interferometer phase in delay units as a function of time and the solid line shows the resulting phase after corrections were applied from the two WVRs. The rms of the corrected phase is approximately 20 ps and corresponds to the expected noise level of the WVRs in this observing mode. Although this is an unusual event on a 7 km we can speculate that it my not be quite so unusual in the uncorrelated atmospheres that one would find using 1000 or 10,000 km baselines. The data indicates that large delay changes are possible in relatively short time periods, and the delay changes are indeed dominated by water vapor. Used properly the WVR is capable of tracking the vapor delay changes with a precision of a few ps.

IV. SUMMARY

Using a simple barometer to measure the surface pressure, a thermometer, something to measure surface water vapor density, and a model, we can estimate the zenith delay and then use any of a half-dozen mapping functions to estimate the delay along the line of sight. If we use a model for the atmosphere that can remove a portion of the dynamics we can achieve a 1 ns delay accuracy at elevation angles of 6 degree. If the experiment is structured properly it is possible to solve for the zenith delay and reduce the atmospheric delay error to less than 1 ns.

If we wish to improve on this capability we must estimate the line of sight vapor delay. An instrument to make accurate measurements of atmospheric brightness temperature at two frequencies near the 22.2 GHz spectral line is called a water vapor radiometer and will cost about $150K. Someone will have to maintain and operate it, and someone will have to analyze the data it produces. For the effort one can anticipate roughly an order of magnitude improvement over models.

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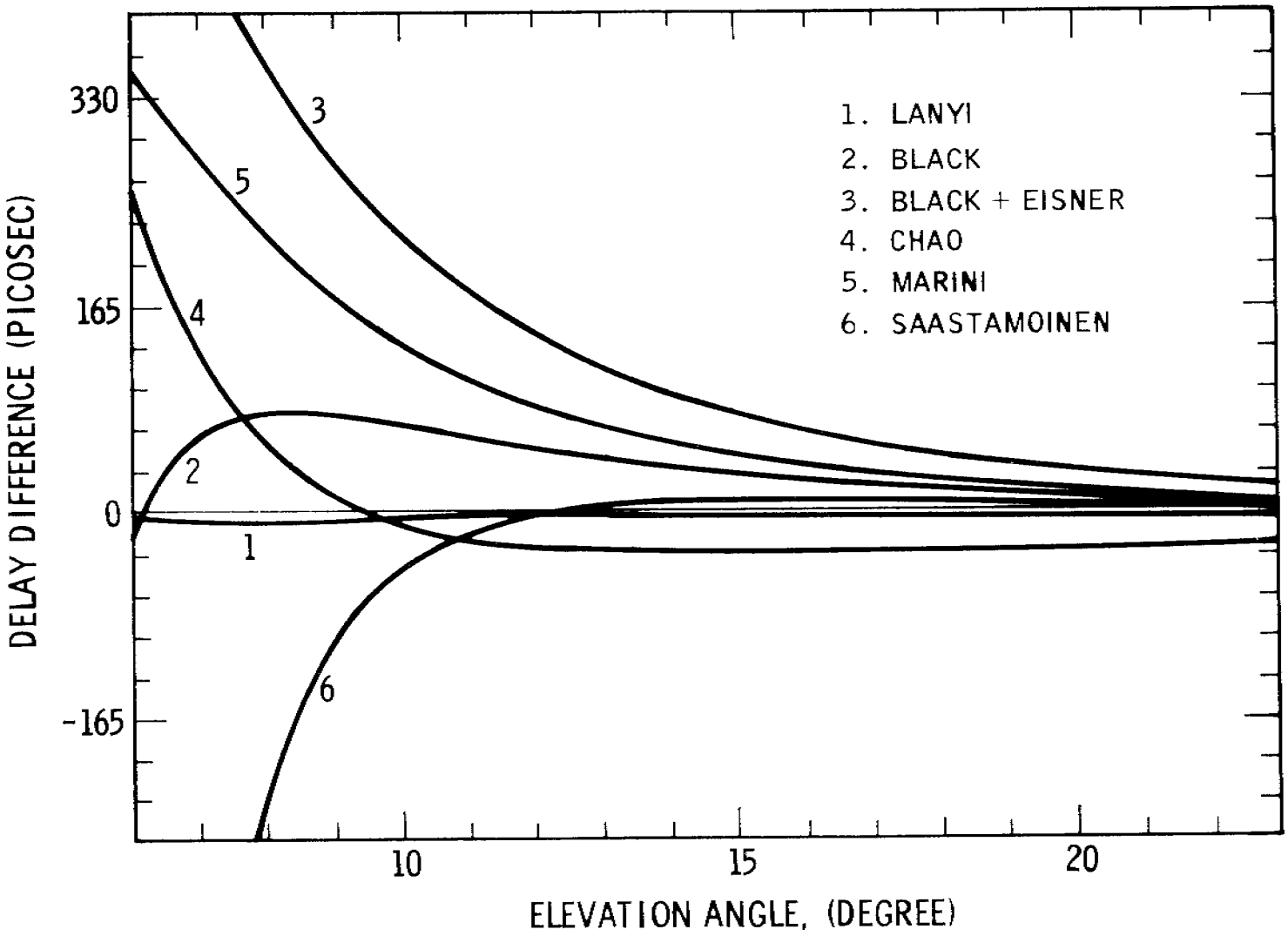

Figure 1 -Difference between the mapping functions and ray tracing

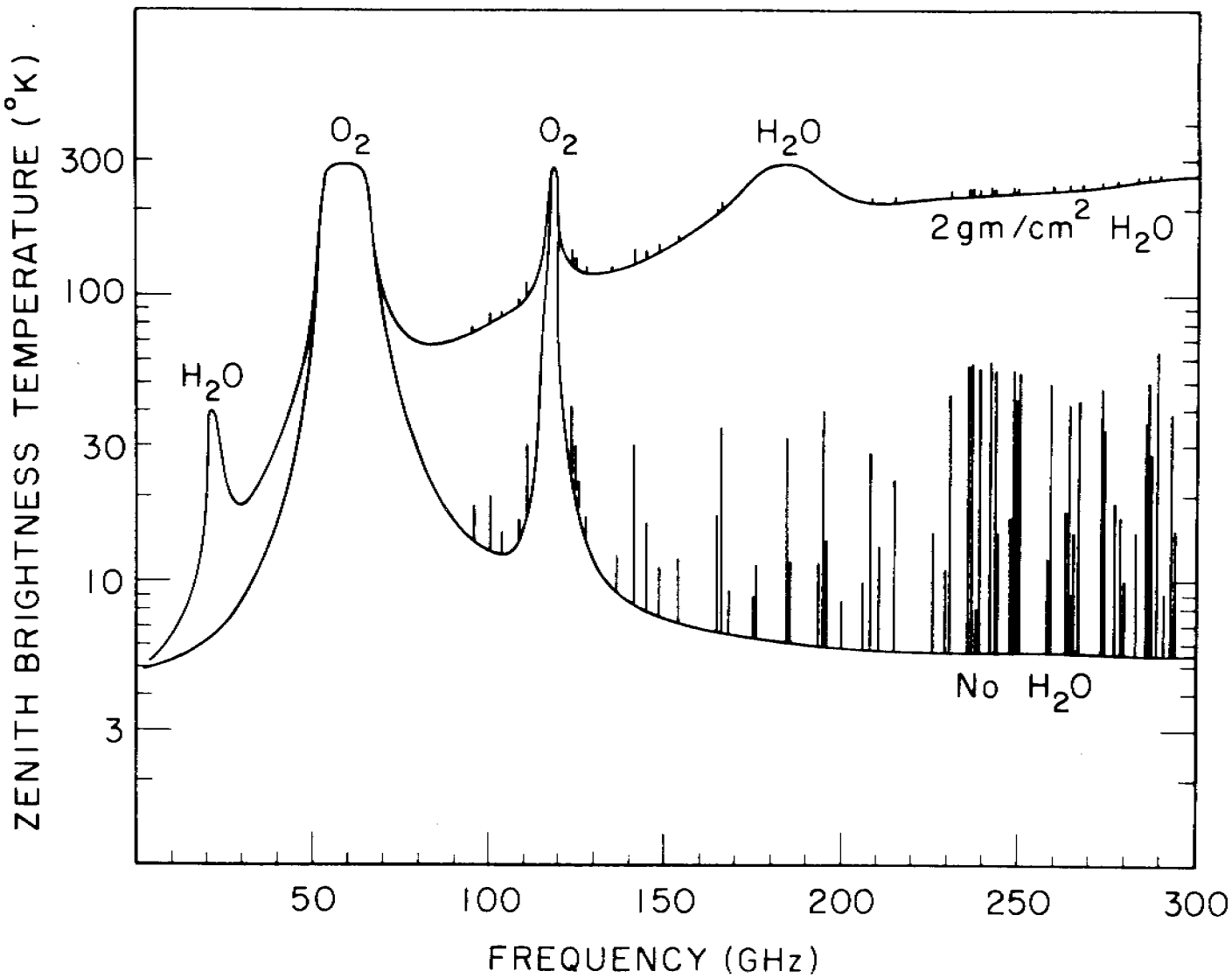

Figure 2 -Brightness Spectrum of the atmosphere

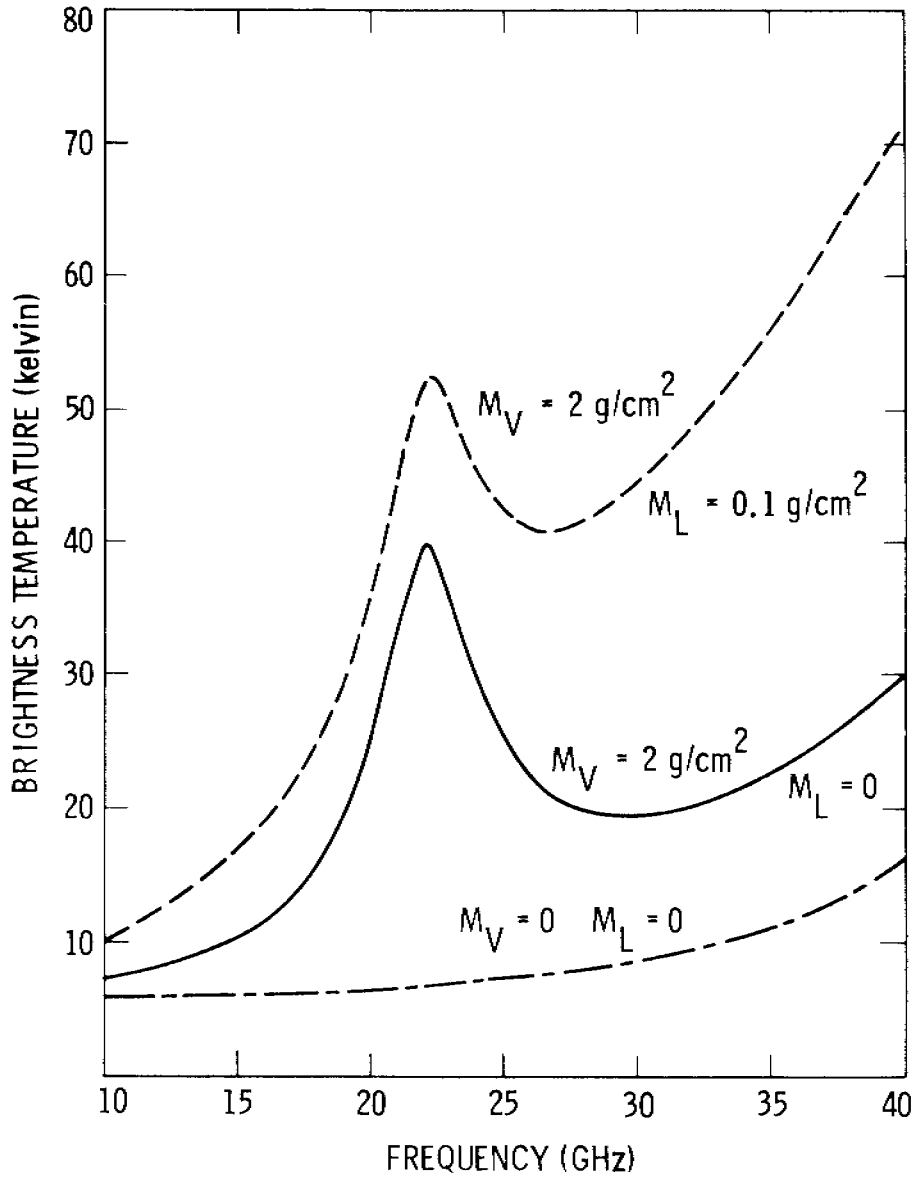

Figure 3 -Effect of liquid water on the brightness temperature

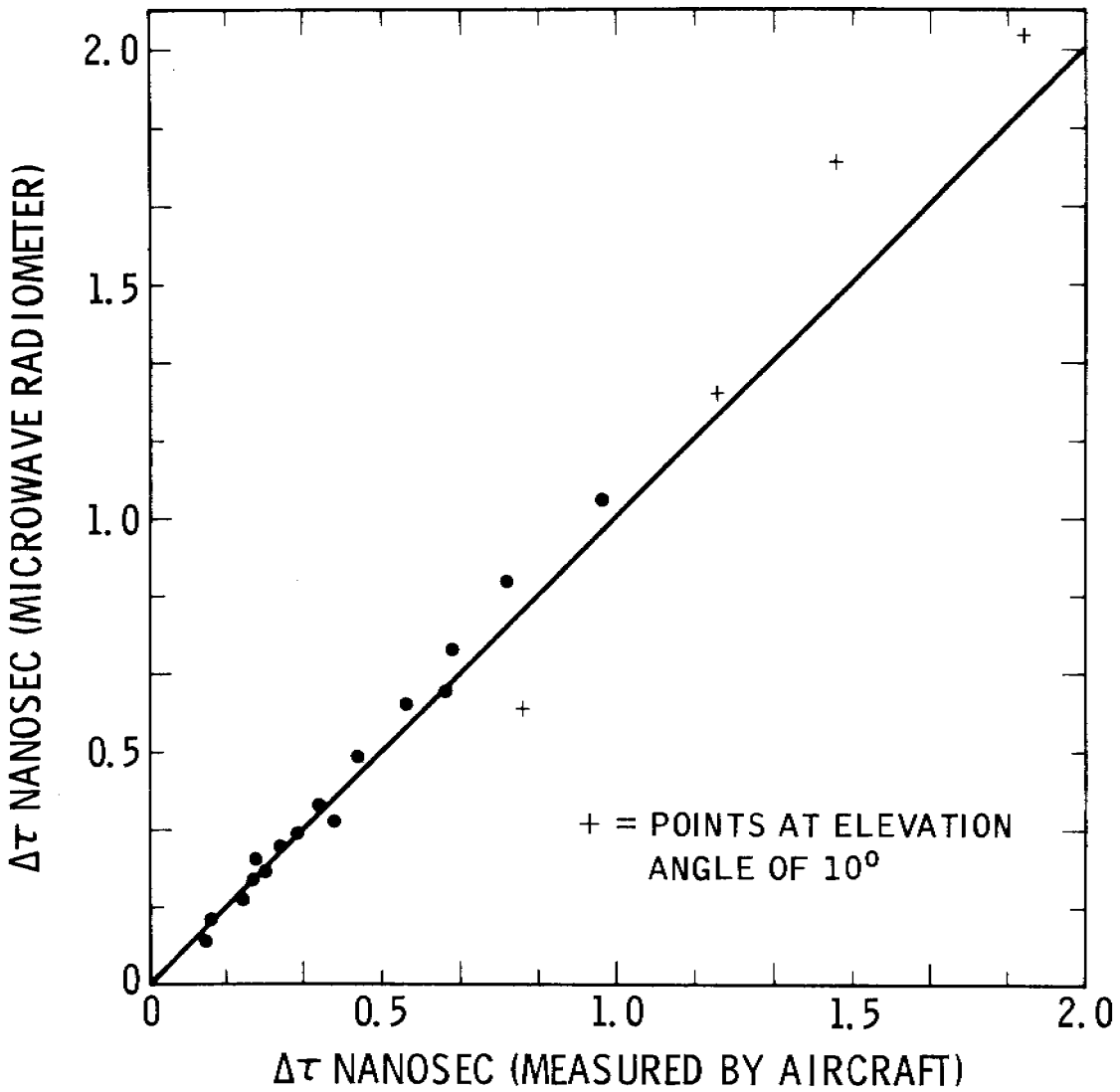

Figure 4 -Accuracy of a water vapor radiometer

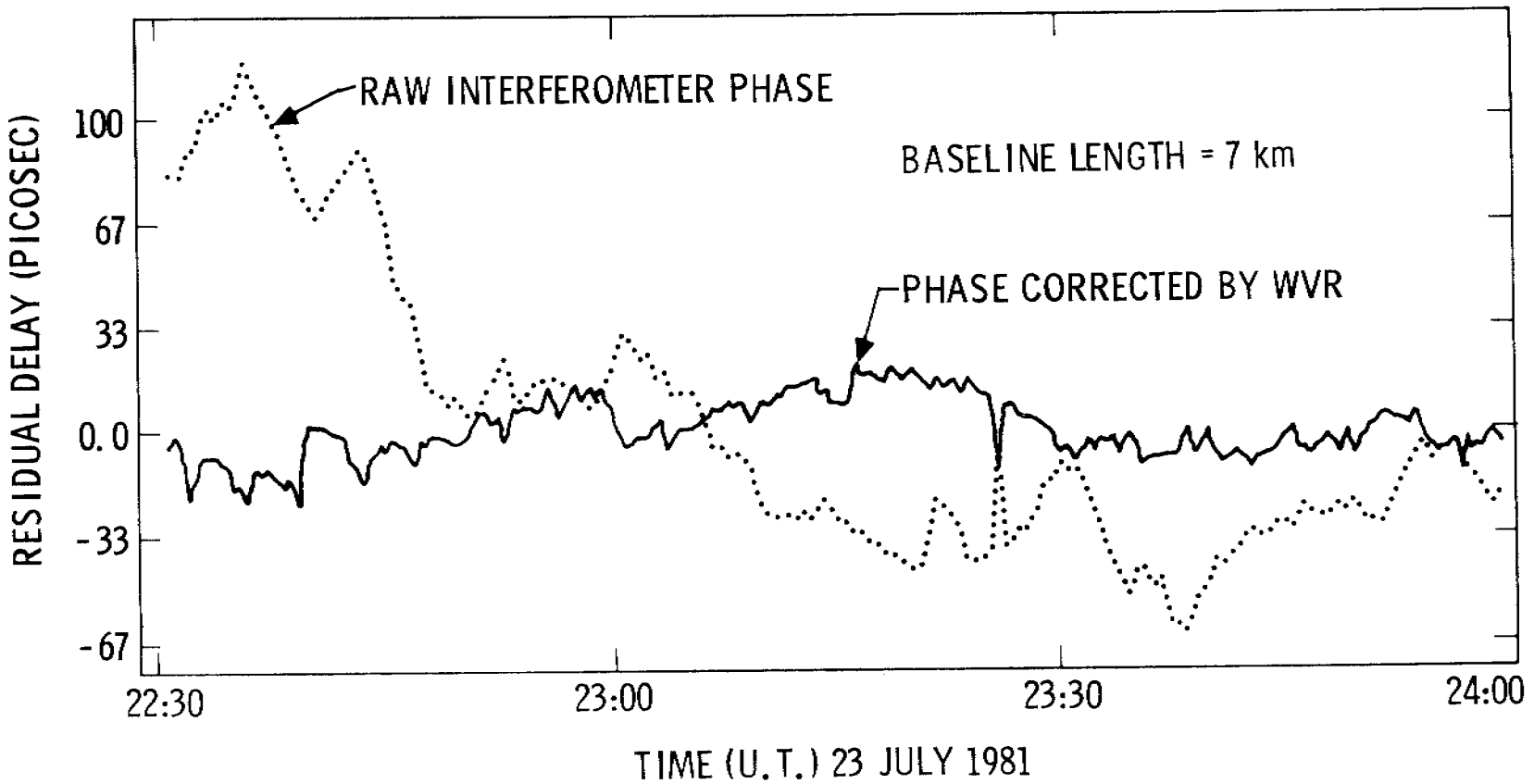

Figure 5 -VLA residual delay before and after correction by WVRs

QUESTIONS AND ANSWERS

NICHOLAS YANNONI: This particular question might be answered best by you, Jack, or perhaps by the speaker. I would like to have a quick comparative statement of the domains of correction that has been addressed by the speakers. These altitude domains, or lines of demarcation where tropospheric effects dominate ionospheric effects. I know that these exist, and would like to have a ballpark statement about them.

MR. KLOBUCHAR: Let me say a few words about the ionosphere. I think that the GPS L-1 frequency is probably a reasonable demarcation line. There are times when the total zenith time delay, due to the ionosphere, might be of the order of a few nanoseconds, say five to ten nanoseconds.

Let me ask, either George or Ed, the zenith time delay due to the atmosphere would be how much?

VOICE FROM AUDIENCE: Nine total.

MR. KLOBUCHAR: About the same. However, they can model theirs. The variability of the troposphere is what, a few percent?

VOICE FROM THE AUDIENCE: Ten percent.

MR. KLOBUCHAR: Is ten percent the highest?

VOICE: That's maximum.

MR. KLOBUCHAR: Whereas the variability of the ionosphere, during the nighttime, when the total delay is five to ten nanoseconds, is very high. It may be 40 or 50 percent. It depends on the region of the world you are in.

That is still about where they become equal. There are times when the ionosphere is several or many tens of nanoseconds at L-1, for instance, and the troposphere never gets to many tens of nanoseconds, does it? I think that you had something like 100 nanoseconds, didn't you, or 100 feet of error?

MR. ALTHSULER: The largest error, right on the horizon, is like 100 meters, but when you get up to four or five degrees, it's more like 100 feet. You are talking about a maximum of 100 nanoseconds, at four or five degrees.

MR. RESCH: It's also strongly frequency dependent. With GPS you have two frequencies, so you have a handle on calibrating the ionosphere to some level, perhaps as good as a few centimeters of equivalent path delay. With the atmosphere, you are left with a model, or a water vapor radiometer as an independent way of coping with the error.

MR. KLOBUCHAR: I guess that the answer is that GPS L-1 is a good ballpark to start arguing. If you get down to a couple of hundred megahertz, the Transit frequencies, then the ionospheric errors probably predominate.

When we get to a few gigahertz, the ionosphere is not so important, although the VLBI people use S and X band to get rid of the ionosphere because it's a relatively easy thing to do. I can't see $150,000 for a dual frequency ionospheric scheme. Certainly around ten gigahertz you start not worrying about the ionosphere, but it's all relative, because a few years ago if you guys could transfer time within a microsecond, everybody was happy. Now you are talking about nanoseconds, and in a few years we will be talking about picoseconds. come back and see us then. The ionosphere won't go away, and I don't think the water vapor and the dry component of the atmosphere will go away either.

MR. KNOWLES: I have just a minor quibble. I think your estimate of 150K for that water vapor radiometer is a bit high. That would certainly decrease when they were made on a production line.

MR. RESCH: I am not so sure about that, at least the quantities. There is at least one company that is making these devices as a commercial product, and in a conversation with one of their representatives a few weeks ago, that was the price that was quoted to me.

MR. PONSONBY, JODRELL BANK, ENGLAND: I would like to ask whether the delays that have been discussed are reciprocal delays? Can we assume that the ionospheric delay in particular is the same for the down path as it is for the up path?

MR. KLOBUCHAR: Yes, period, and also for Faraday rotation. It's very interesting that back when people first started measuring it, some people thought that you would get rotation in one direction for the up-going signal and rotation in the other direction for the down-going signal and thus get cancellation of the rotation. You folks at Jodrell Bank did some of the early work in that and know that you get twice the amount. The paths are essentially identical, at least for the frequencies that we are talking about.