Fundamental Concepts and Definitions in PM and AM Noise Metrology

TUTORIAL – QUESTIONS AND ANSWERS

Note from the editor

The questions were asked at various points during the presentation. They were transcribed and are presented here at the end of each tutorial.

JIM COMPARO (AEROSPACE CORP.): So S_v is the power spectrum density of that full voltage signal?

EVA PIKAL (NIST): Yes.

JIM COMPARO (AEROSPACE CORP.): And the first you said was what?

EVA PIKAL (NIST): The carrier.

JIM COMPARO (AEROSPACE CORP.): I see three terms there. One is contribution due to the phase noise; one is a contribution to the amplitude noise; and then there's a term out in front. And what is that?

EVA PIKAL (NIST): That's just a carrier, right? That's – you know, if it were ideal, it would just be a delta function at the frequency of oscillation.

JIM COMPARO (AEROSPACE CORP.): I guess my question is – and maybe I'm getting way ahead, but if there is some correlation between the amplitude noise and the phase noise, then the power spectrum of the voltage wouldn't necessarily be symmetric, would it? And so would it be fair to sort of consider these things as folded over on top of one another?

EVA PIKAL (NIST): I believe this assumes there is a correlation between AM noise and PM noise in the signal.

MARC A. WEISS (NIST): I am looking at "requires a reference of comparable stability." I thought you said we could use the oscillator under test as a reference as well.

EVA PIKAL (NIST): That's to measure the noise floor. You need a different reference to measure phase noise of the test oscillator. You need another oscillator. To measure the noise floor, you need to use the single oscillator to get rid of the noise of the source and the reference.

II. DISCUSSION OF ERROR MODELS FOR PM AND AM NOISE MEASUREMENTS

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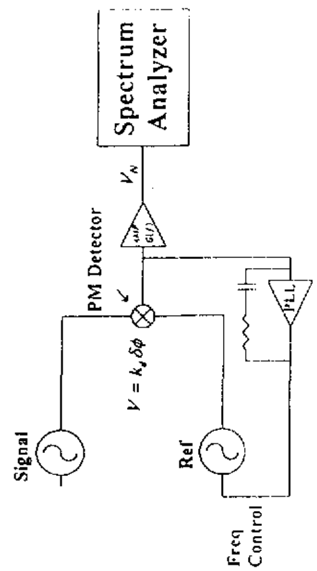
A. Error model for PM noise measurements

B. Error model for AM noise measurements

C. PM and AM noise models

D. Conversion of PM data to $\sigma_y(\tau)$ and mod$\sigma_y(\tau)$

Simple PM Measurements



$\frac{PSDV_N}{[k_d G(f)]^2}$ measures $S_\phi(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low PM noise. Results uncorrected for PLL and gain variations with Fourier frequency.

ERROR MODEL FOR PM MEASUREMENTS

1. DETERMINATION OF K
2. DETERMINATION OF AMPLIFIER G(f)
3. PLL EFFECTS (IF ANY)
4. CONTRIBUTION OF AM NOISE
5. HARMONIC DISTORTION
6. CONTRIBUTION OF SYSTEM NOISE FLOOR
7. CONTRIBUTION OF REFERENCE NOISE
8. STATISTICAL CONFIDENCE OF DATA
9. LINEARITY OF SPECTRUM ANALYZERS
10. ACCURACY OF PSD FUNCTION

1. DETERMINATION OF K

TRANSDUCER SENSITIVITY DEPENDS ON

A. Frequency

B. Signal power and impedance, reference power and impedance

C. Mixer termination at all three ports

D. Cable lengths

ACCURACY OF DETERMINATION DEPENDS ON DEGREE ABOVE PARAMETERS HELD CONSTANT PLUS

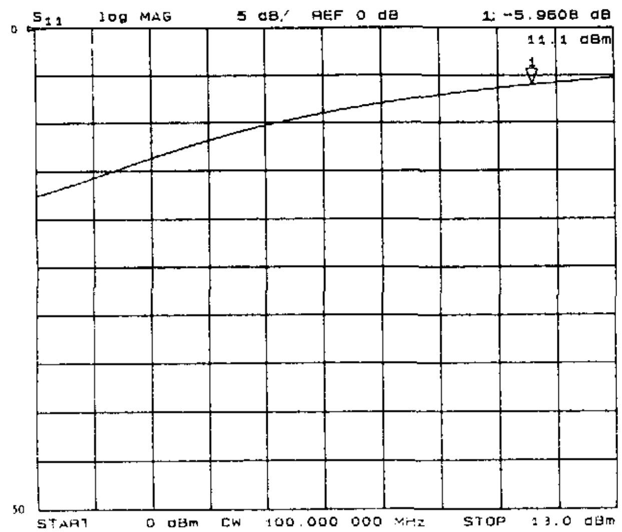
A. Symmetry of waveform

B. Signal-to-noise-ratio

C. Phase deviation from 90°-depends on noise level, dc offset-may depend on f

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

[illegible]
A LO OF 15 dBm



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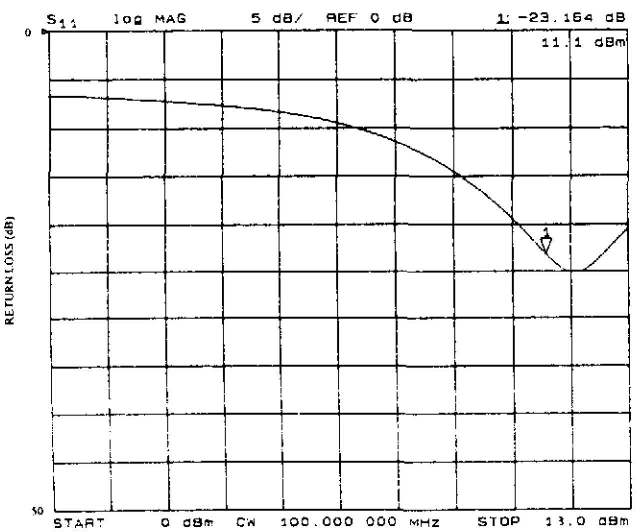
MIXER B RETURN LOSS VERSUS RF POWER AT 100 MHZ AND
A LO OF 15 dBm



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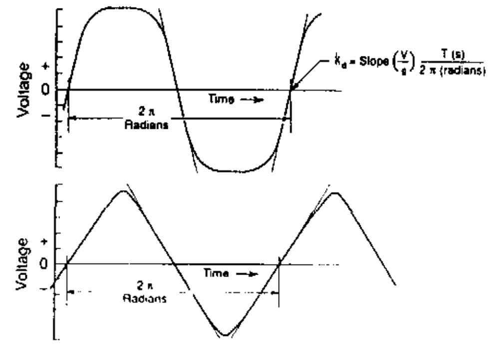
MIXER OUTPUT VOLTAGE VERSUS PHASE (TIME)



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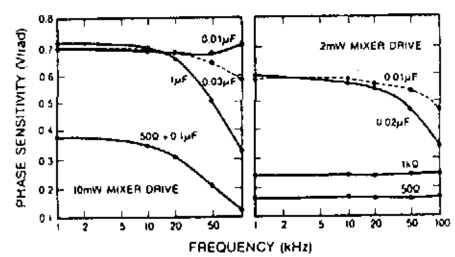
MIXER SENSITIVITY Kd VERSUS IF LOAD



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2. DETERMINATION OF AMPLIFIER GAIN VERSUS FOURIER OFFSET

G(f) DEPENDS ON

A. Intrinsic amplifier G(f)

B. Mixer output impedance

C. Signal power, impedance, and cable length through B.

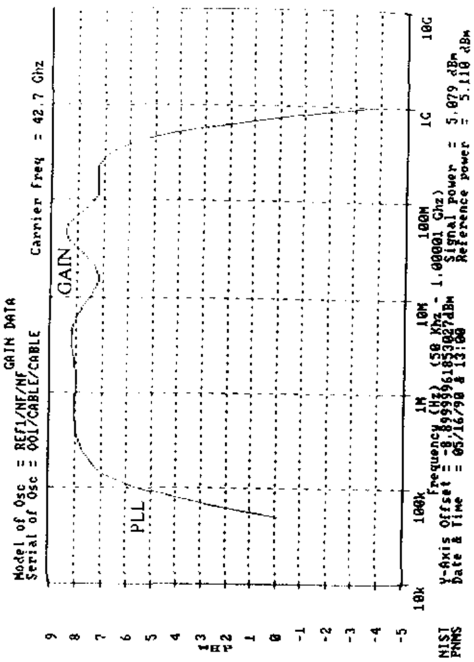
E. Reference power, impedance, and cable length through B.

ACCURACY OF DETERMINATION DEPENDS ON THE DEGREE ABOVE PARAMETERS HELD CONSTANT PLUS

A. Linearity and slewing rate of amplifier

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

PLL AND GAIN EFFECTS ON G(f) Kd



3. PLL EFFECTS (IF ANY)

ATTENUATION OF THE LOW FREQUENCY PHASE DEVIATION CAN BE REDUCED BY

A. Normal PLL loop. Results may be altered by additional filters in electronic frequency control (EFC) path

B. Signals that propagate through the power sources of the two oscillators

C. Signals that propagate through the air to pull the frequency of one or both signals

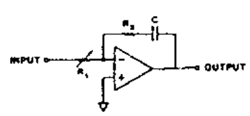
E. Signals that propagate through the measurement system (mixer) to pull the frequency

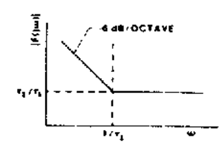
F. Injection lock feedback from the cavity discriminator or delay line discriminator

PLL EFFECTS SHOULD BE MEASURED IN SITU SINCE MANY EFFECTS IN THE EFC PATH ARE HIDDEN.

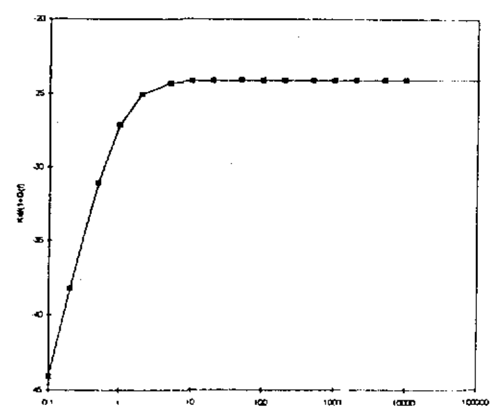
ERRORS IN PARAMETERS 1-3 ARE OFTEN CORRELATED

PLL RELATIONS

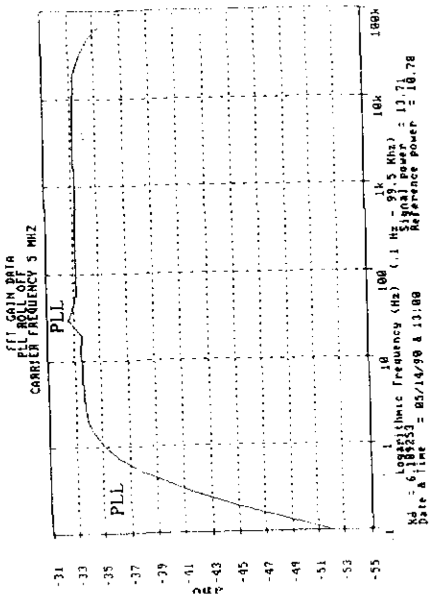




$$G(f)_{PLL} = C\frac{(1 + j\omega R_2 C)}{j\omega R_1 C} \qquad V_d = \frac{K_d(\Delta\phi_{test} - \Delta\phi_{ref})}{1 + G(f)_{PLL}}$$



PLL EFFECTS ON G(f) Kd



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MEASUREMENTS OF $S_\phi(f)$ @ MHz SYNTHESIZER VS OSCILLATOR

f (Hz)	$S_\phi(f)\|_{AB}$ (db/Hz)	$S_a(f)\|_{AB}$ (dB/Hz)	$S_a(f)\|_A$ $\beta^2 A$ (dB/Hz)	Measured Noise Floor dB rel Rad^2/Hz	Actual Noise Floor
32	-119.8	-126.0	≈ -151.0	-154.0	-160.0
100	-124.2	-127.0	≈ -152.0	-154.0	-165.0
1 K	-132.1	-132.0	≈ -157.0	-158.0	175.0
10 K	-137.3	-133.0	≈ -158.0	-158.0	175.0
100 K	-136.8	-133.0	≈ -158.0	-158.0	175.0

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4. CONTRIBUTION OF AM NOISE

AM TO PM CONVERSION IS UNIVERSAL

A. Occurs via non-linear process

B. Typically -15 to -25 dB in double balanced mixers

C. Can reach - 3 dB in some amplifiers

D. Sets the noise floor in many measurements

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5. HARMONIC DISTORTION

A. Harmonics of signal and reference contribute to K and detected noise

B. PM noise on harmonics may not be same as fundamental

C. Sensitivity depends on power, impedance, harmonic number

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TO GET NOISE FLOOR SET A = B

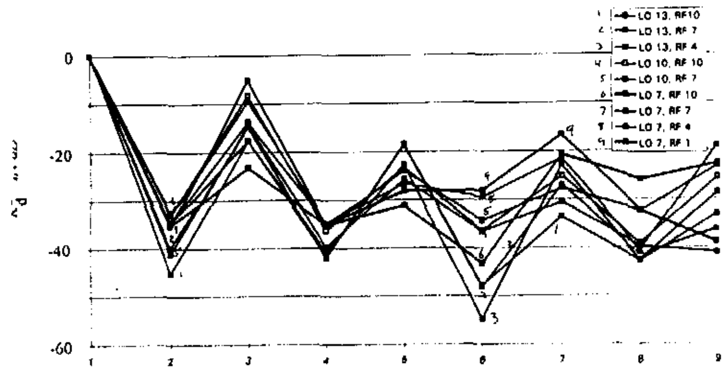
$$S_\phi(f)_{Noise\ Floor} \approx S_{\phi A}(2\pi f\tau_{delay})^2 + \frac{V_n(f)^2_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_\phi(f)_{power\ splitter}$$

$$(2\pi f\tau_{delay})^2 S_\phi(f) = (\frac{\pi}{20})^2 S_\phi(f) \quad for \quad f \approx \frac{v}{10}, \tau_{delay} = \frac{\pi}{2}$$

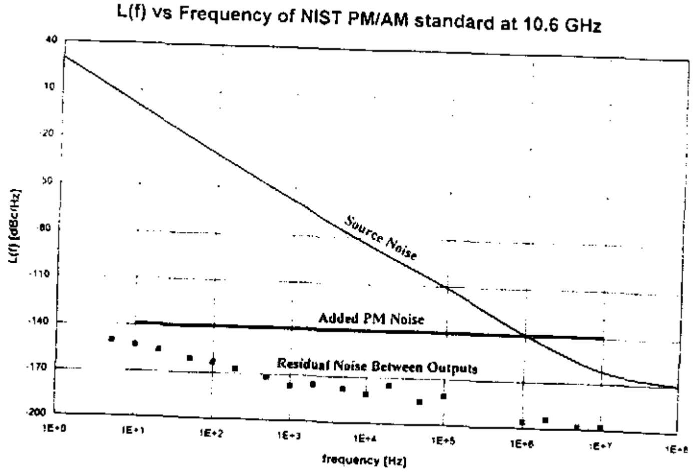
TO CALCULATE INDIVIDUAL PM NOISE FOR AN OSCILLATOR

$$S_\phi(f)_{AB} + S_\phi(f)_{AC} - S_\phi(f)_{BC} = 2S_{\phi A}(f) + \frac{V_n^2}{K_d^2 BW} + 2S_{aA}(f)\beta_A^2$$

HARMONIC SENSITIVITY OF MIXER VS RF AND LO POWER IN dB



L(f) vs Frequency of NIST PM/AM standard at 10.6 GHz



6. CONTRIBUTION OF SYSTEM NOISE FLOOR

NOISE TERMS INCLUDED IN $\frac{PSD(V_n)}{K_d^2 G(f)^2}$

$$S_\phi(f) = \frac{[\Delta\phi_A(f) - \Delta\phi_B(f)]^2}{BW} + \frac{V_n(f)^2_{mixer}}{K_d^2 BW} + \frac{V_n(f)^2_{amp}}{K_d^2 BW}$$

$$+ \frac{V_n(f)^2_{SA}}{K_d^2 G(f)^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

$$S_\phi(f)_{pair} = S_{\phi A}(f) + S_{\phi B}(f) + \frac{V_n(f)^2_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

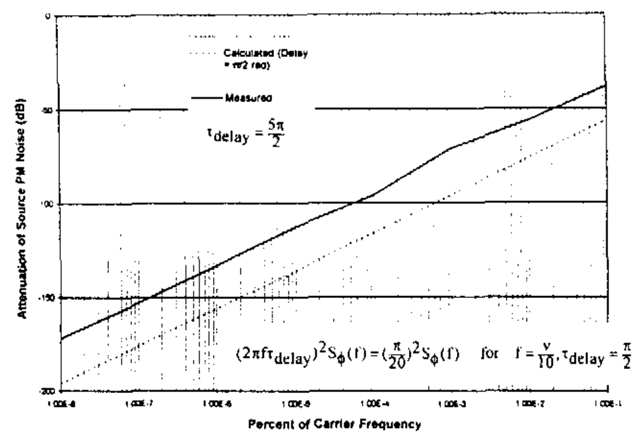
8. STATISTICAL CONFIDENCE OF THE DATA

Table 1. Approximate : 68% confidence Intervals for FFT Spectral Estimates N > 10

power law noise type	uniform	window Hanning	flattened peak
f^0	$1.02/\sqrt{N}$	$0.98/\sqrt{N}$	$0.98/\sqrt{N}$
f^{-2}	$1.02/\sqrt{N}$	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$
f^{-3}	unusable	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$
f^{-4}	unusable	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$

$$S = S_m\left(1 \pm \frac{B}{\sqrt{N}}\right)$$

Phase Noise added by delay line at 10.6 GHz Carrier Frequency



7. CONTRIBUTION OF REFERENCE AM AND PM NOISE

NOISE TERMS INCLUDED IN $\frac{PSD(V_n)}{K_d^2 G(f)^2}$

$$S_\phi(f) = \frac{[\Delta\phi_A(f) - \Delta\phi_B(f)]^2}{BW} + \frac{V_n(f)^2_{mixer}}{K_d^2 BW} + \frac{V_n(f)^2_{amp}}{K_d^2 BW} + \frac{V_n(f)^2_{SA}}{K_d^2 G(f)^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

$$S_\phi(f)_{pair} = S_{\phi A}(f) + S_{\phi B}(f) + \frac{V_n(f)^2_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

STATISTICAL UNCENTAINTY OF FFT SPECTRAL DENSITY MEAUREMENTS

$S_m(f) = S(f)\,[1 \pm k/N^{x}]$

$k = 1 \rightarrow 68\%$, $k = 1.9 \rightarrow 95\%$ CONFIDENCE $N \geq 10$

N = number of samples averaged

Number of Samples	k = 1 (approx. 68%) $S_m = S[1\pm\delta]$, $S_m \,^{\gamma}_{-\beta}$ dB			k = 1.9 (approx. 95%) $S_m = S[1\pm\delta]$, $S_m \,^{\gamma}_{-\beta}$ dB		
	δ	γ	β	δ	γ	β
4	0.54	-2.	+3.3	2.5	-3.	+6
6	0.42	-1.5.	+2.3	1.4	2.5.	+5
10	0.32	-1.2.	+1.7	0.61	-2.1.	+4
30	0.18	-0.72.	+ .86	0.35	1.3.	+1.8
100	0.1	-0.41.	+0.46	0.19	-0.76.	+0.92
200	0.058	-0.24.	+0.25	0.14	-0.46.	+0.51
1000	0.032	-0.13.	+0.13	0.06	-0.26.	+0.28
3000	0.018	-0.08.	+0.08	0.035	-0.15.	+0.15
10000	0.01	-0.04.	+0.04	0.019	-0.08.	+0.08

D. B. Percival and A.T. Walden, "Spectral Analysis for Physical Application," Cambridge Univ. Press, 1993

B. N. Taylor and C. E. Kuyatt, NIST Technical Note TN1297 1993

STATISTICAL UNCENTAINTY OF SWEPT RF SPECTRAL DENSITY MEAUREMENTS

$S_m(f) = S(f)\,[1 \pm k\,(VIDEO_{bw}/NRES_{bw})^{½}]$

$k = 1 \rightarrow 68\%$, $k = 1.9 \rightarrow 95\%$ CONFIDENCE $N \geq 10$

$VIDEO_{bw}$ = video bandwidth

N = number of sweeps averaged

RES_{bw} = resolution bandwidth ≤ f/10

$\frac{NRES_{bw}}{VIDEO_{bw}}$	k = 1 (approx. 68%) $S_m = S[1\pm\delta]$, $S_m{}^{\gamma}_{\beta}$ dB			k = 1.9 (approx. 95%) $S_m = S[1\pm\delta]$, $S_m{}^{\gamma}_{\beta}$ dB		
	δ	γ	β	δ	γ	β
4	0.54	-2 ,	+3.3	2.5	-3 ,	+6
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9. LINEARITY OF SPECTRUM ANALYZER

A. Accuracy of wide dynamic range

B. Digitizing errors

C. Need to segment spectrum with filters

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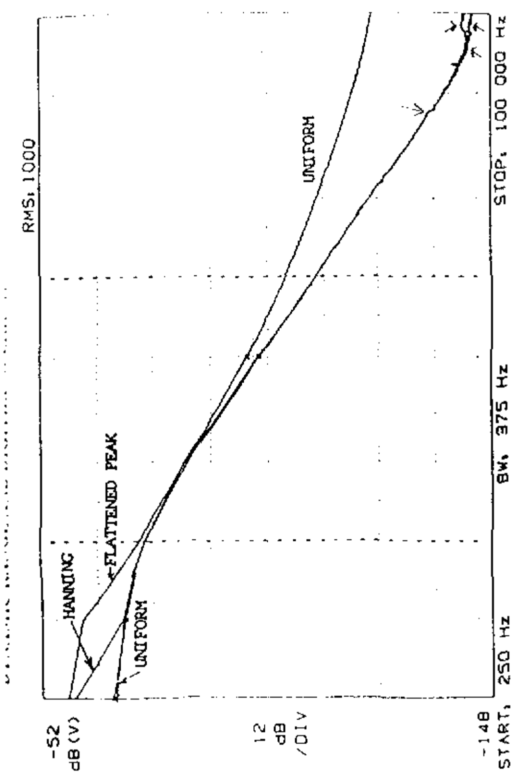

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10. ACCURACY OF THE PSD FUNCTION

DEPENDS ON

A. Signal type

Use flat top window for bright lines

Use Hanning window for noise

B. Window function and Fourier frequency (leakage)

f should be less than span/23 for Flat top window

f should be less than span/75 for Flat top window

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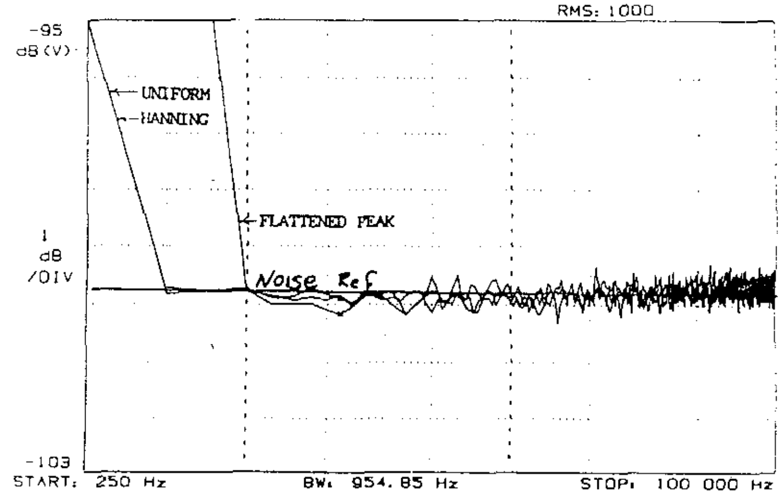
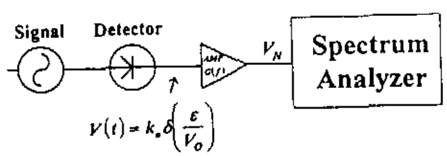
APPROXIMATE BIASES IN FFT SPECTRAL DENSITY ESTIMATORS

	Noise Type f^0			Noise Type f^4		
Channel #	Flat Top	Hanning	Uniform	Flat Top	Hanning	Uniform
1	20.1 dB	19.6 dB	19.6 dB	10.0 dB	8.6 dB	Not Useable
2	16.7	Small	Small	9.1	0.4	
3	7.22	↓	↓	4.0	0.4	
4	Small			1.2	Small	
5	↓			1.1	↓	
6				1.1		
7				1.0		
8				0.8		
9				0.6		
10				0.6		
11				0.5		
12				0.4		
13				0.4		
14				Small		
15				↓		

PSD OF f^0 NOISE



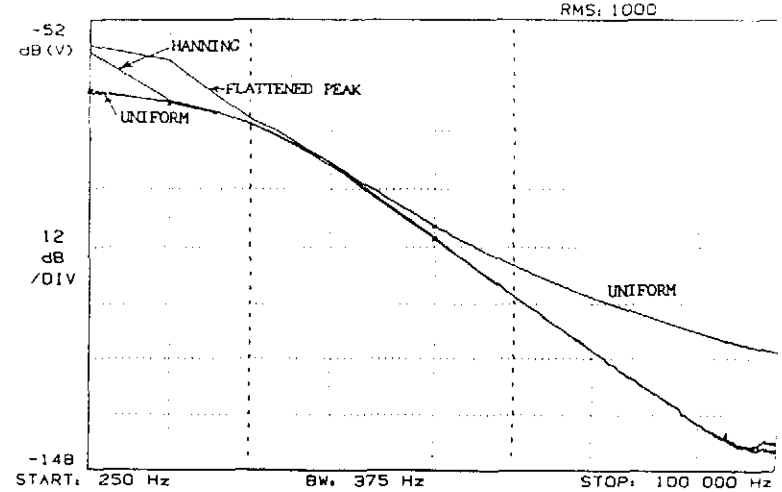
Simple AM Measurement



$\frac{PSDV_N}{[k_o G(f)]^2}$ measures $S_a(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low AM noise.

PSD OF f^4 NOISE



ERROR MODEL FOR AM MEASUREMENTS

1. DETERMINATION OF K
2. DETERMINATION OF AMPLIFIER G(f)
3. CONTRIBUTION OF SYSTEM NOISE FLOOR
4. STATISTICAL CONFIDENCE OF DATA
5. *LINEARITY OF SPECTRUM ANALYZERS*
6. ACCURACY OF PSD FUNCTION

1. DETERMINATION OF Ka

DETECTOR SENSITIVITY DEPENDS ON

A. Carrier frequency

B. Signal power and impedance

C. Detector termination both ports

D. Cable lengths

E. Fourier frequency

Sensitivity to Fourier frequency is often difficult to measure due to bandwidth of most AM modulators

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

2. DETERMINATION OF AMPLIFIER G(f)

Depends on

A. Detector output impedance

B. Signal power, impedance, and cable length through A

C. Fourier frequency

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

3. CONTRIBUTION OF AM SYSTEM NOISE FLOOR

A. *Noise floor difficult to measure in single channel systems*

B. Cross-correlation can be used to determine noise floor (part III)

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

MODEL FOR PM IN AMPLIFIERS

PM

$$S_\phi(f) = \left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] \Rightarrow S_a(f)$$

AM — NEW

LEESON'S MODEL FOR PM IN OSCILLATORS

$$S_\phi(f) = \left(\frac{\upsilon_0}{2Q_L}\right)^2 \frac{1}{f^2}\left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] + \left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] + \left(\frac{\upsilon_0}{2Q_L}\right)^2 \frac{1}{f^2}\left[\frac{\alpha_R}{f}\right]$$

	Amplifier		Resonator
$BW = \upsilon_0/2Q_L$	f<BW	f>BW	f<BW

NOISE MODEL OF AMPLIFIERS

AM and PM similar 1/f + thermal

NOISE MODEL OF OSCILLATORS

PM complicated-see examples

PM typically includes $1/f^3$ + thermal

AM depends on circuit and degree of limiting

AM sometimes 1/f + attenuated thermal

NOISE MODEL OF PM MEASUREMENT SYSTEMS

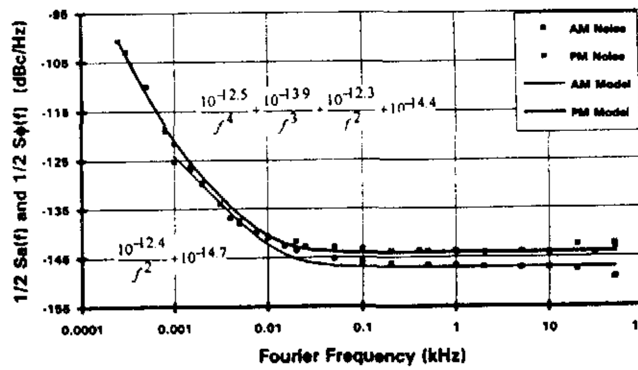
1/f + thermal for two oscillator type

$1/f^3$ + thermal for single oscillator type

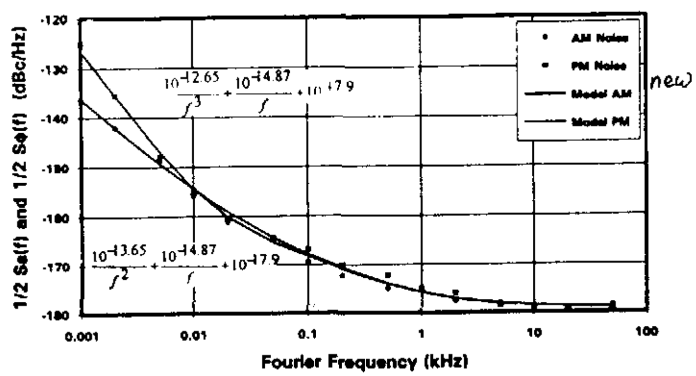
NOISE MODELS OF AM DETECTORS

1/f + thermal

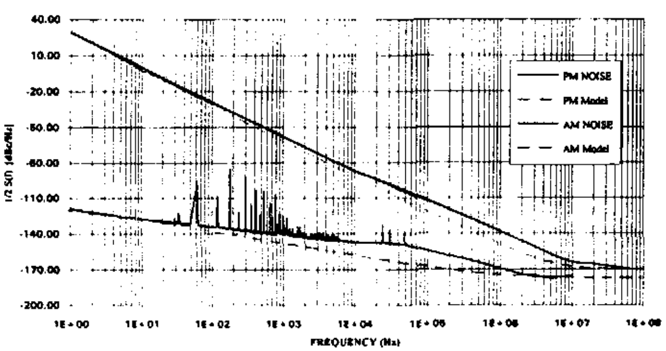
5 MHz AM and Phase Noise



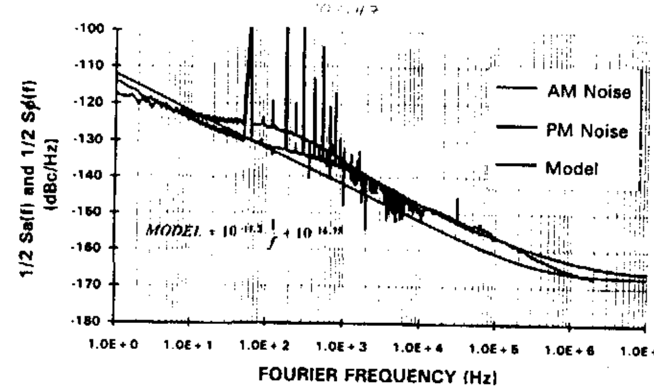
5 MHz AM and PM Noise



new

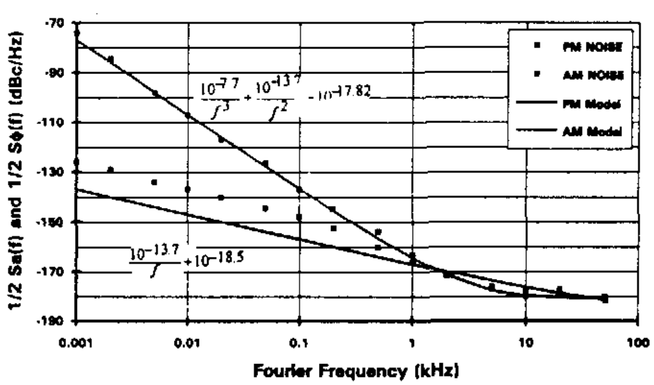


AM AND PM NOISE IN AMPLIFIER #2

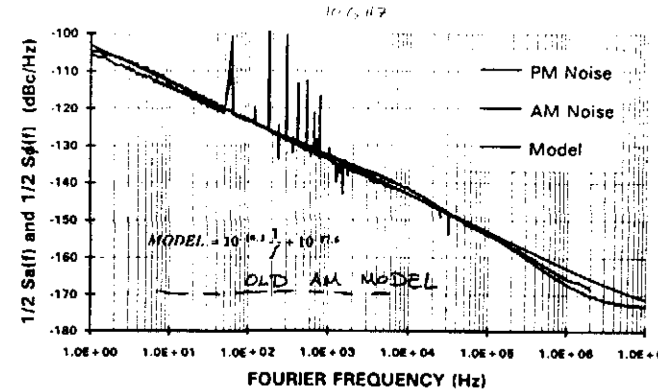


$$\frac{1}{2}S_\phi(f) = \frac{1}{2}S_a(f) = \alpha_E\frac{1}{f} + \frac{2kTFG(f)}{P_o}$$

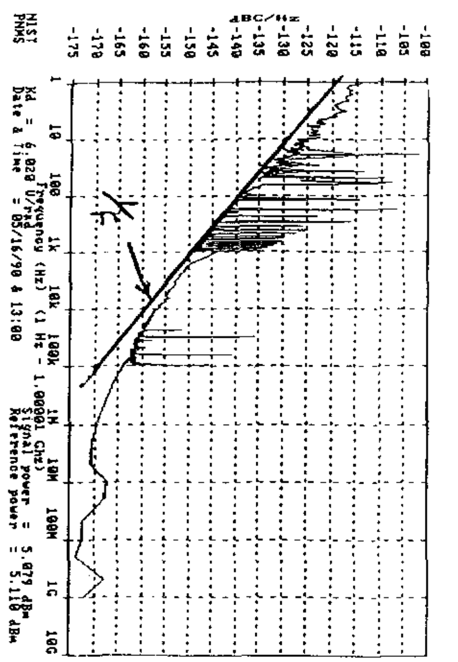
100 MHz AM AND PHASE NOISE



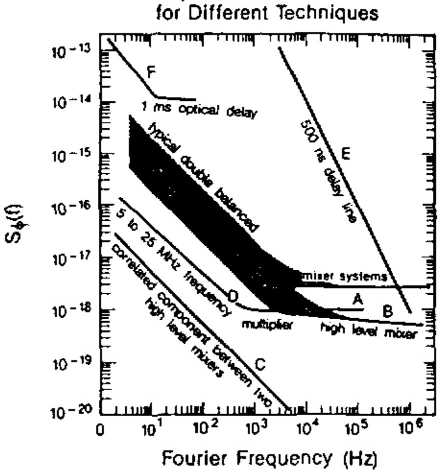
AM AND PM NOISE IN AMPLIFIER #1



$$\frac{1}{2}S_\phi(f) = \frac{1}{2}S_a(f) = \alpha_E\frac{1}{f} + \frac{2kTFG(f)}{P_o}$$



NOISE FLOOR OF WIDE-BAND NIST PM MEASUREMENT SYSTEM AT 42 GHz



PHASE NOISE RELATIONSHIPS

$$S_\phi(f) = \mathcal{L}(\nu_o - f) + \mathcal{L}(\nu_o + f)$$

$$dBC/Hz = 10 \log \mathcal{L}(f)$$

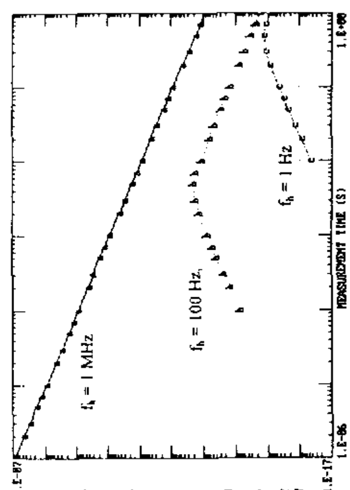
$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f) \quad rad^2/Hz \quad 0<f<\infty$$

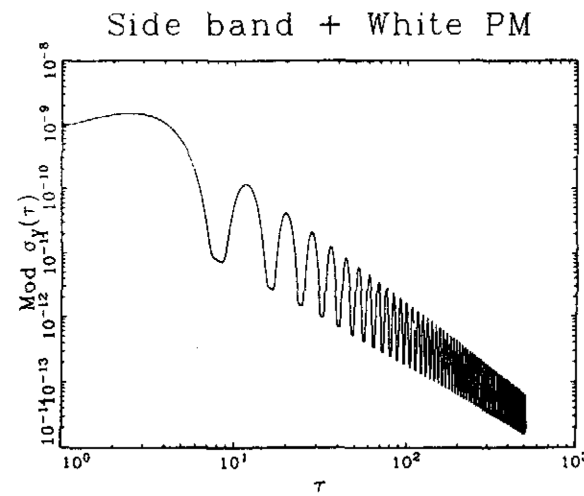
$$\sigma_y^2(\tau) = 2\int_0^\infty df\, S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}$$

$$Mod\ \sigma_y(n\tau_o) = \left(\frac{2}{n^2(\pi n \tau_o)^2} \int_0^{f_h} S_y(f)\, \frac{\sin^6(\pi f n \tau_o)}{f^2 \sin^2(\pi f \tau_o)}\, df \right)^{1/2}$$

CONVERSION OF $S_\phi(f)$ TO $\sigma_y(\tau)$ FOR

$S_\phi(f) = \frac{4X10^{-16}}{f} + 1X10^{-17}$ AT 100 MHz




Side band + White PM
Mod $\sigma_y(\tau)$
τ