TIME DOMAIN FREQUENCY STABILITY ESTIMATION BASED ON FFT MEASUREMENTS

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Abstract

The standard characterizations of frequency stability are, in the time domain, the Allan (or two-sample) variance and, in the frequency domain, the spectral density function (SDF). The former is mathematically related to the latter by the conversion between time and frequency domain. In this paper, the biases of the Fast Fourier transform (FFT) spectral estimate with Hanning window are checked and the resulting unbiased spectral density are used to calculate the Allan variance. Both the numerical integral and the curve-fitting methods are presented to calculate the variances. The numerical integral is a straightforward method to use, and we can get the integral approximation after eliminating some spike points from SDF, e.g. noise caused by ac power. In addition, a common model for SDF is linear combinations of power-law processes, which are distinguished by the integer powers in their functional dependence on Fourier frequency with the appropriate coefficients. Fitting a form of the above model to the resulting SDF using standard regression techniques can estimate these coefficients. Cutler's formula is adopted to calculate the integral approximation using these coefficients. The approximations of variances from these two methods are compared and analyzed. Finally, we discuss the limitations and possible errors from these two methods.

INTRODUCTION

FFT spectrum analyzers generally have several different window functions available for analyzing signals. Recent research has shown that the Hanning window provides excellent performance for analyzing noise **[1]**. Our lab has established a phase noise measurement system including a phase noise standard (1,5,10,100 MHz), a single-channel noise detector, a delay line unit, and one single-channel FFT spectrum analyzer. The signal reference is from a low-noise frequency reference (LNFR-400) with a noise level of about –173 dBc/Hz (5 MHz PM, at Fourier frequency 100 KHz). For passive devices, the system can measure up to –177 dBc/Hz. This year we also have built up a cross-correlation system, which can measure the noise 20 dBc/Hz below the above level. The very short-term stability ($\tau < 0.5$ second) by using this phase noise measurement system is a subject of interest to us, since the traditional time interval counter is applicable only when τ is about 1 second. In this paper, we are trying to calculate the Allan variance of the spectral density estimate from experimental results, and possible errors in the spectral density estimate, e.g. biases from window functions, noise from ac power, etc. will be considered. In general, if the spectral density of the normalized frequency fluctuations $S_y(f)$ is known, its mathematical relation to the Allan variance can be expressed as **[2]**:

$$\sigma_y^{2}(\tau) = 2\int_0^{f_h} S_y(f)\frac{\sin^4(\pi\tau f)}{(\pi\tau f)^2}df$$

(1)

where f_h is the high frequency cutoff of a low-pass filter. The Allan variance can be easily calculated using numerical integration while experimental data are properly processed. Besides, the power-law model is frequently used for describing phase noise, and it assumes that the spectral density of fractional frequency fluctuations is equal to the sum of terms, each of which varies as an integer power of Fourier frequency. Thus, there are two quantities that completely specify $S_y(f)$ for a particular power-law process: the slope on a log-log plot for a given range of f and the amplitude. The slope is denoted by α and therefore f^{α} is the straight line on a log-log plot that relates $S_y(f)$ to f. The amplitude is denoted by h_α and hence **[2-3]**:

$$S_y(f) = \begin{cases} \sum_{\alpha=-2}^{+2} h_\alpha f^{\alpha} & \text{for } 0 < f < f_h \\ 0 & \text{for } f > f_h \end{cases}.$$

(2)

Cutler derived equation (3) from equation (1) and (2):

$$\sigma_y^{2}(\tau) = h_{-2}\frac{(2\pi)^2}{6}\tau + h_{-1}2\ln 2 + \frac{h_0}{2\tau} + h_1\frac{1.038 + 3\ln(2\pi f_h\tau)}{(2\pi)^2\tau^2} + h_2\frac{3f_h}{(2\pi)^2\tau^2}$$

(3)

The value for each coefficient h_α could be obtained using standard regression techniques, and then we have the Allan variance. Details for practicing these methods are discussed in the following sections.

THE EXPERIMENTS

The measurement system consists of a FSS1000E noise detector, a FSSM100 phase noise standard, a FSS1011A delay line unit, a SDI LNFR-400 low-noise frequency reference, and one SRS-760 FFT spectrum analyzer with a Hanning window adopted. The measurement procedures and data recording are automated under the software TestStation version 3.0.

The basic and simple experiment is the system noise floor test. The LNFR-400 5 MHz output is split with a reactive splitter to provide two input signals. These signals with either of them passing through the noise standard in advance are connected to the phase noise detector. The output of the noise detector is then fed into the FFT spectrum analyzer. A composite graph with both the calibration line and the corrected phase noise data is shown in Figure 1. The noise components of 60 Hz, 120 Hz, 180 Hz, etc. are commonly encountered because ac power is getting into the measurement system or the source under test. The use of a phase noise standard greatly speeds up the process of calibrating the noise measurement system, and we can quickly review the calibration of the system while looking at the final data.

CALCULATION OF EXPERIMENTAL RESULTS

Because biases due to linearity and SDF accuracy of FFT spectrum analyzers are typically less than 0.4 dB with 95% confidence limits, we may temporarily neglect their influence in the process of calculating Allan variance. In Figure 2, the raw data of the above-mentioned noise floor test and the ones with outliers removed are shown. Notice that most of the outliers are from the noise of ac power. In Figure 3, we calculate the Allan deviation σ_y (square root of Allan variance) of these two spectral data using the numerical integration techniques with sampling time τ = 0.1 s ~ 10 s.

Note that the amplitude of Allan deviation of the raw data varies up and down depending on the sampling time, while the other doesn't vary obviously. This is because in the time domain the sensitivity to a periodic wave varies directly as the sampling interval. This effect (which is an alias effect) is a very powerful tool for filtering out a periodic wave imposed on a signal source **[4]**. Also found is that although the influence of ac power noise is important in calculating Allan deviation, eliminating these outliers from the raw data seems to reveal the true characteristic of the device under test (DUT) reasonably.

Next, we will calculate the Allan deviation of the spectral data with outliers removed using the curve-fitting methods and compare the results with the ones using numerical integration method. In the frequency domain, $L(f)$ is the prevailing measure of phase noise among manufacturers and users of frequency standards, and it is defined as:

$$S_y(f) = \frac{f^2}{\nu_0^2} S_\phi(f) \tag{4}$$

$$\mathrm{L}(f) = \frac{1}{2} S_\phi(f) \tag{5}$$

$$\frac{dBc}{Hz} = 10\log(\mathrm{L}(f)) \tag{6}$$

In Figure 1, it is a $L(f)$ vs. f plot with its x-y axis in log scale. For f = 1 Hz ~1000 Hz, we see that when f increases by one decade, $L(f)$ also goes down by one decade. This noise process can be identified as flicker PM. For f = 10 ~ 99.75 kHz, we have white PM. We calculate the $S_y(f)$ first and use the functions $S_y(f) = h_1 f^1$ and $S_y(f) = h_2 f^2$ to fit the data in flicker PM and white PM region respectively, and then get h_1 = 4.68×10^{-28} and h_2 = 2.61×10^{-31}. This regression technique is called as the polynomial curve-fitting method. The power-law processes can also be expressed as:

$$\ln S_y(f) = \ln h_\alpha + \alpha \ \ln f \qquad (\alpha = -2 \ldots +2) \tag{7}$$

We can also fit the data in log-log space with α = 1 and α = 2 and then get h_1 = 4.07×10^{-28} and h_2 = 2.68×10^{-31}. This regression technique is called as the log scale curve-fitting method. Figure 4 shows the calculated Allan deviation using Cutler's equation with these coefficients, while the sampling time τ is from 0.1 to 10 s with an increment of 0.1 s. We find that the results using both the curve-fitting methods are very close, but they also keep an obvious bias from the ones using the numerical integration method. The former are roughly about twice as large as the latter for the whole sampling interval. For example, the $\sigma_y(\tau = 0.1s)$ using curve-fitting methods is about 4.80×10^{-13}, while the one using numerical integration is 2.24×10^{-13} and, when τ = 10 s, the related values are about 4.94×10^{-15} and 2.46×10^{-15}.

CONCLUSIONS

In this paper, we calculate and compare the Allan deviation of the experimental spectral results using the numerical integral and curve-fitting methods. We find that the influence of ac power noise play an important role in calculating Allan deviation, so eliminating these outliers from the raw data seems to reveal the true characteristic of the DUT reasonably. As for the resulting bias between curve-fitting and numerical integration methods, we have tried to shift the spectral estimate in the range of 10 dB based on the assumption that possible errors may occur in the calibration process and checked the bias, but it still exists without improvement. In order to solve this problem, we will do more research in the near future.

REFERENCES

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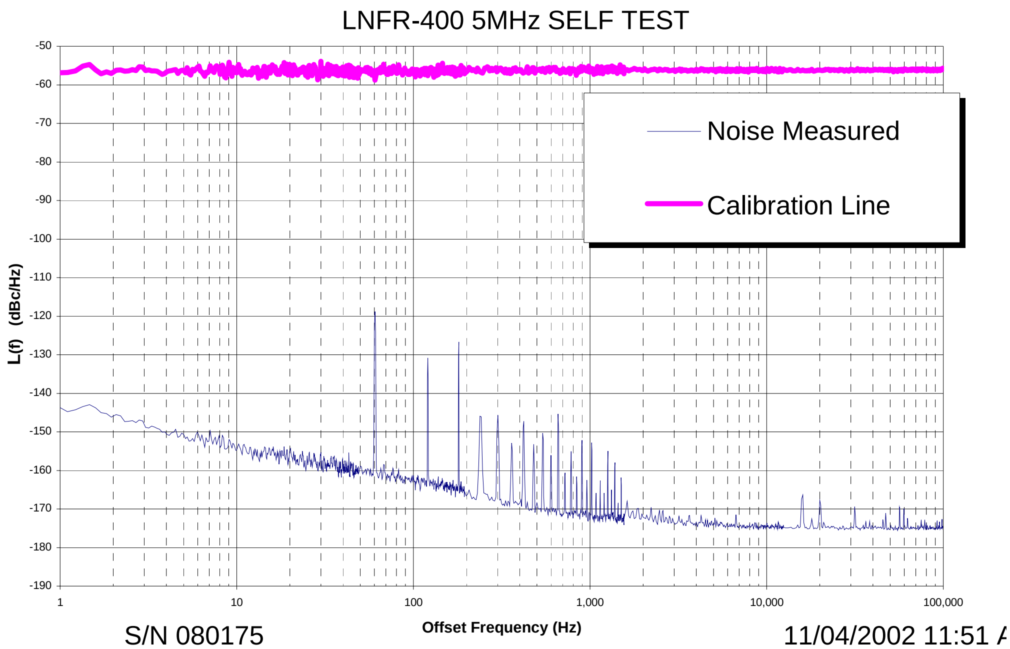

Figure 1. Phase Noise Measurement of System Noise Floor (5 MHz).

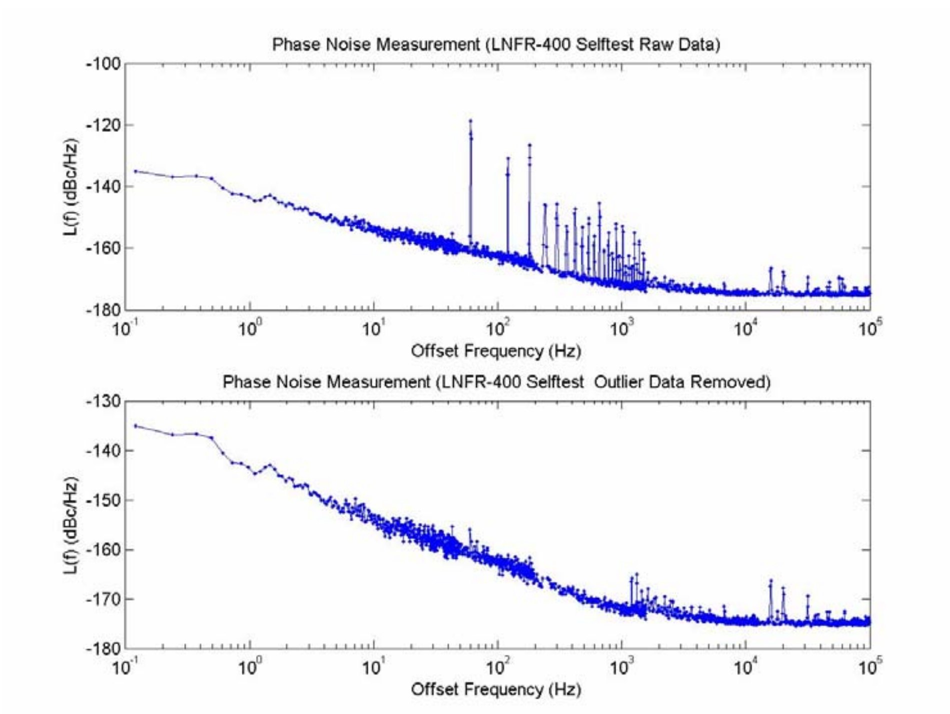

Figure 2. Raw Data and Data with Outliers Removed.

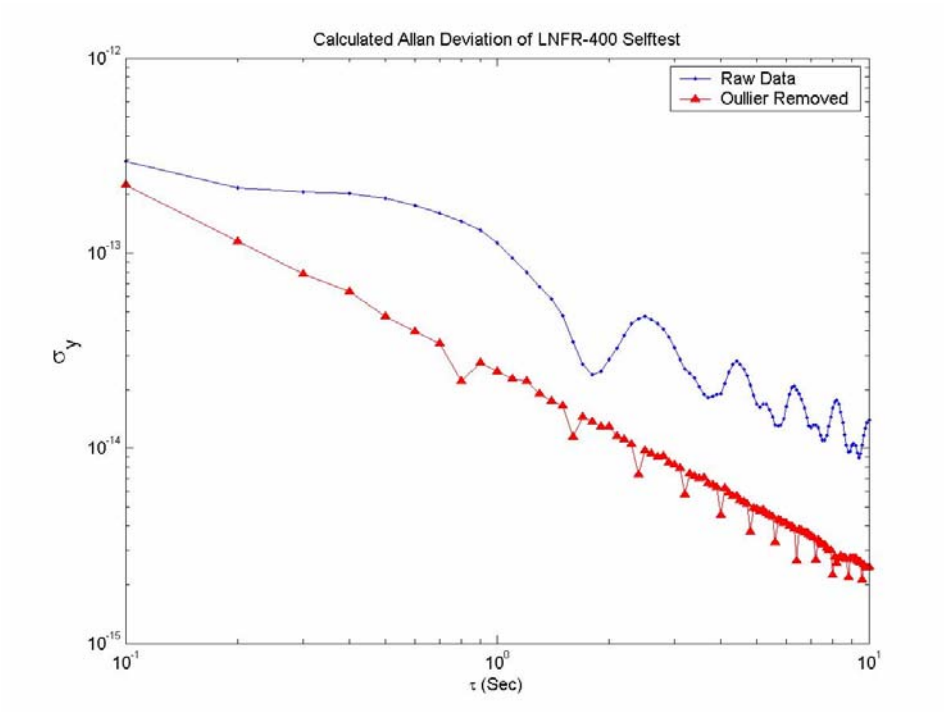

Figure 3. Calculated Allan Deviation of Two Spectral Data.

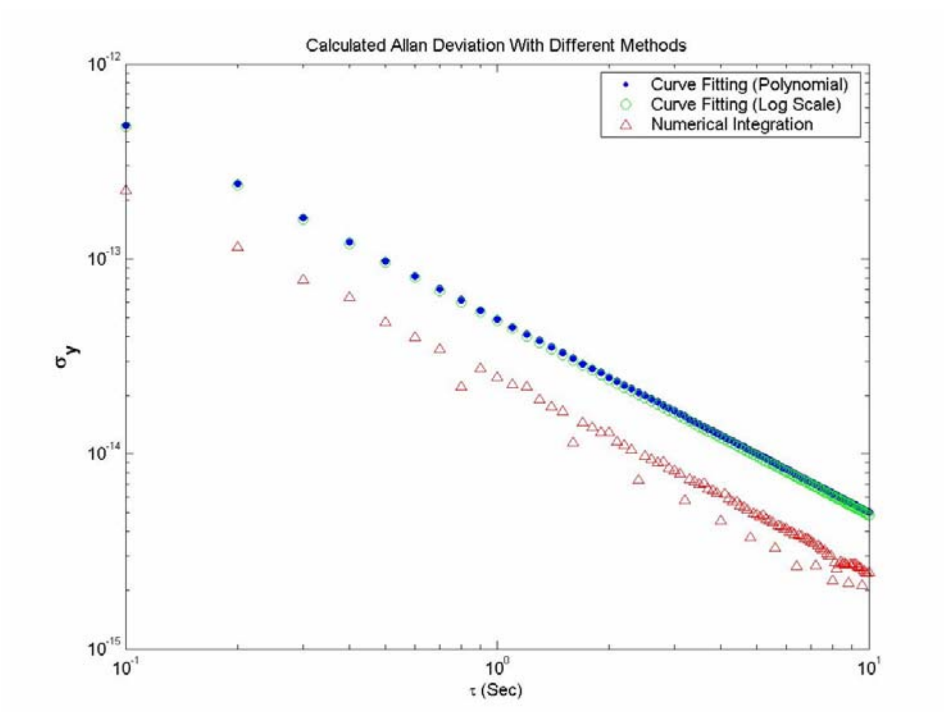

Figure 4. Calculated Allan Deviation with Different Methods.