

# Study on GPS Common-view Observation Data with Multiscale Kalman Filter based on correlation Structure of the Discrete Wavelet Coefficients

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**Abstract**—In this paper, we pay our attention to the multiscale kalman algorithm based on correlation structure of the discrete wavelet coefficients for the restoration of the GPS common-view observation data. Based on the hypotheses of that the GPS common-view observation data being pretreatment possess of 1/f fractal characteristics. In this condition, we estimate the Hurst parameter of GPS clock difference data based upon the wavelet transform. When  $0 < H < 1$ , the GPS clock difference data is taken for as a Gaussian Zero-mean nonstationary stochastic process which can be considered having the 1/f fractal characteristics. So, we can talk about the correlation structure of the discrete wavelet coefficients. During the course of the estimation of the GPS common-view data with the multiscale kalman bank, we process the single-channel and multi-channel common-view observation data, respectively. Comparisons between which results and circular T demonstrate our algorithm's feasibility and effectiveness.

## I. INTRUCTION

It must be remarked that the conclusions obtained in the paper is based upon the following hypotheses:

(1) The GPS common-view observation data after pretreatment possess of 1/f fractal characteristics. A model for this process called fractional Brownian motion (fbm), has been proposed by Manderbrot and vanness. And then, the parameters estimation of 1/f fractal signal is turned into the estimation of Hurst parameters of fbm.

(2) To consider the GPS common-view clock difference data processes is demonstrated by  $y_H(t)$ ,  $t \in IR$ , while, the

increment  $G(t) = y_H(t+1) - y_H(t)$  is discrete fractal Gaussian noise. This discrete Gaussian noise is a Zero-mean stationary Gaussian random stochastic process. We analyze this discrete fractal Gaussian noise by wavelet transform and based

upon the variance equation of the least square algorithm, which is used to educe the method of estimating the Hurst parameter, this Hurst parameter is one of the parameter of GPS common-view clock differences data.  $0 < H < 1$  is demonstrated that these discrete data is a Gaussian Zero-mean nonstationary stochastic process which can be considered having the 1/f fractal characteristics.

In this paper, According to [1], the wavelet transform and the least square algorithm are used to estimate our Hurst parameter of the GPS data. In paper [2], [3], [4], the correlation structure of the wavelet coefficients sequence are talked about. Based on these characteristics, the multiscale kalman filter bank is designed. In section II, we expatiate the wavelet transform algorithm of how to estimate Hurst parameter. In section III, on the base of Hurst exponent, the correlation structure of the wavelet coefficients sequence are discoursed in every scale. Furthermore, in section IV, the multiscale kalman filter based on the better mean-square estimation is obtained. Finally, in section V, the single-channel and multi-channel GPS common-view observation data are processed by the filters, respectively. We give out the experiment results.

## II. THE ESTIMATION OF EXPONENT H OF GPS CLOCK DIFFERENCE DATA

In the paper, we suppose the GPS common-view data after pretreatment possess of the 1/f fractal signal character, based on this hypothesis, we can figure out its exponent H. if  $0 < H < 1$ , the condition of our algorithm is satisfied (that is, the GPS common-view data after pretreatment possess of the 1/f fractal signal character ), then the algorithm talked about, in the paper, can be used to estimate our clock difference data. According to [1], we use the Mallat algorithm to analyze discrete fractal Gaussian noise of the discrete clock difference data. Wavelet transform and the least square algorithm are used to estimate the exponent H. According to Mallat , In [5]

$$a_m[k] = \sum_{j=-\infty}^{\infty} h_{2k-j} a_{m-1}[k], x_m[k] = \sum_{j=-\infty}^{\infty} g_{2k-j} a_{m-1}[k] \quad (1)$$

While  $g_k$  and  $h_k$  are the filter coefficients of wavelet function  $\phi(t)$  and scale function  $\varphi(t)$ , respectively. And  $g_k = (-1)^k h_{1-k}$ . Haar wavelet is the most simply orthonormal basis. For Haar wavelet, there is  $h_0 = h_1 = g_0 = -g_1 = \sqrt{2}$ , with this, (1) turned into

$$\begin{aligned} a_{m+1}[k] &= 2^{-1/2}(a_m[2k] + a_m[2k-1]) \\ x_{m+1}[k] &= 2^{-1/2}(a_m[2k] - a_m[2k-1]) \end{aligned} \quad (2)$$

According to the theorem of [6], we obtain the “approximate” wavelet coefficients  $a_m[k]$  and “detail” wavelet coefficients  $x_m[k]$  by transforming discrete fractal Gaussian noise on the bases of orthonormal Haar discrete wavelet basis according to Mallat algorithm. Let  $R_m[j]$ ,  $V_m$  are the autocorrelation of the wavelet coefficients and the variance of the “detail” wavelet term, respectively. Then

$$R_m[j] = 2^{(2H-1)m} R[j], \quad (3)$$

$$V_m = 2^{(2H-1)(m-1)} \delta_H^2 (2 - 2^{2H-1}) \quad (4)$$

Here, we think the wavelet transformation is a whitening filter for fractal discrete Gaussian noise, which have maximally reduced the correlation of original signal. So, the correlation of wavelet coefficients obtained by wavelet transform can be ignored. Because the mean value of fractal discrete Gaussian noise is zero and the mean value of the wavelet coefficients after wavelet transform is also zero, the variance  $\text{Var}(x_m^k)$  of “detail” wavelet term resulted from transforming fractal discrete Gaussian on the bases of Haar basis according to Mallat algorithm can be estimated by the following equation:

$$\delta_m^2 = \frac{1}{N(m)} \sum_{k=1}^{N(m)} (x_m^k)^2 \quad (5)$$

While  $N(m)$  is the number of detail wavelet coefficients in the  $m$ th scale. According to (4), we can obtain:

$$\delta_m^2 = 2^{rm} \delta_H^2 (2 - 2^r) \quad (6)$$

while  $r=2H-1$ , the unknown parameters wait for us to estimate is  $(\delta_H^2, r)$ .

We get the new nonlinear equation by operating logarithm on both sides of (6) equation.

$$\alpha_m = \beta + r_m + \log_2(2 - 2^r) \quad (7)$$

Here,  $\alpha_m = \log_2 \delta_m^2$ ,  $\beta = \log_2 \delta_H^2$ .

$$\text{Let } s(\beta, r) = \sum_{m=1}^M (\alpha_m - \beta - r_m - \log_2(2 - 2^r))^2 \quad (8)$$

The least square estimation of  $(\beta, r)$  is obtained by the

least  $\left( \hat{\beta}, \hat{r} \right)$  in  $s(\beta, r)$ . So,

$$\begin{cases} \frac{\partial s}{\partial \beta} = \sum_{m=1}^M (\alpha_m - \beta - r(m-1) - \log_2(2 - 2^r))^2 = 0 \\ \frac{\partial s}{\partial r} = \sum_{m=1}^M m (\alpha_m - \beta - r(m-1) - \log_2(2 - 2^r))^2 = 0 \end{cases} \quad (9)$$

We can get

$$\hat{r} = \left[ 12 \sum_{m=1}^M m \alpha_m - 6(M+1) \sum_{m=1}^M \alpha_m \right] / (M(M^2 - 1)) \quad (10)$$

$$\text{H results from } \hat{H} = (\hat{r} + 1) / 2 \quad (11)$$

We can obtain by experiment that, with the increase of the wavelet decompose scale, the estimation precision will be relevant increased. But the increase of the wavelet decompose scale cannot obviously improve the estimation precision. Meanwhile, not the more decompose scale imply the more precision of the estimation results; but with the increase of the number of the experiment points, the precision of the estimation results will be corresponding improved.

### III. CORRELATION ANALYSIS

We have detail deduced the estimation process of Hurst parameters. Next, we will analyze the correlation of wavelet coefficients about what data we are interested. In [3], We know that the 1/f –type fractal processes are the slow decay of the correlation structure associated with long -term dependencies. For Haar wavelet, it possesses of better correlation within scale than other wavelets, the dependency information between scales and within scale will be better used to improve the performance of signal process. In this paper, the Haar wavelet is used for further study.

To consider our discrete clock difference data is demonstrated by  $y_H(k)$ , from [5], the wavelet series of the process  $y_H(k)$  up to the scale j is given by

$$y_H(k) = 2^{-j/2} \sum_{n=0}^{(N_0/2^j)} a_j[n] \phi(2^{-j}k - n) + \sum_{j=1}^J 2^{-j/2} \sum_{n=0}^{(N_0/2^j)-1} d_j[n] \psi(2^{-j}k - n) \quad k=0, \dots, N_0-1. \quad (12)$$

According to [3], [7], the approximation term  $a_j[n]$  of the wavelet expansion can be avoided,  $y_H(k)$  can be approximately denoted by

$$\hat{y}_H(k) = \sum_{j=1}^J 2^{-j/2} \sum_{n=0}^{(N_0/2^j)-1} d_j[n] \psi(2^{-j}k - n) \quad (13)$$

We consider the Haar wavelet

$$\psi(t) = \begin{cases} +1, & 0 \leq t < 1/2 \\ -1, & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{from which the orthonormal}$$

basis of the multiresolution analysis is obtained. According to [3], after some calculation, it is possible to obtain the autocorrelation function of the sequence of wavelet coefficients for  $n \neq 0$

$$R_j(n) = E[d_j(m+n)d_j(m)] = \frac{\delta^2}{2(2H+1)(2H+2)} \xi[n](2^j)^{2H+1} \quad (14)$$

With

$$\xi[n] = \left[ |n-1|^{2H+2} - 4|n-(1/2)|^{2H+2} + 6|n|^{2H+2} + |n+1|^{2H+2} - 4|n+(1/2)|^{2H+2} \right] \quad (15)$$

according to [3] and our validation, from (14), (15), we conclude that with the Haar wavelet and  $H=1/2$ , the sequences of wavelets coefficients in each scale j are uncorrelated, but when  $H \neq 1/2$ ,  $R_j(n) \neq 0, |n| > 1$ , which demonstrate the wavelet coefficients is correlated.

The variance of the wavelet coefficients is given by

$$R_j(0) = \text{var}(d_j[n]) = \frac{\delta^2}{2} V_\psi(H) (2^j)^{2H+1} \quad (16)$$

$$\text{With } V_\psi(H) = \frac{1-2^{-2H}}{(H+1)(2H+1)} \quad (17)$$

In the next section, we will use Haar multiresolution analysis and the former equation to design our multiscale filters to estimate our clock difference data.

#### IV. BANK OF KALMAN FILTERS BASED ON CORRELATION STRUCTURE OF WAVELET TERM SERIES

To consider the real value of clock difference at the time of K is demonstrated by  $\mathcal{X}_k$ , which constitute the state variable  $X_k$ . That is,

$$X_k = (\mathcal{X}_k),$$

The observation data is

$$y(k) = H(k)X(k) + w(k)$$

Where,  $H(k)$  is observation matrix,  $X(k)$  is the data of clock difference to be estimated.  $w(k)$  is the observation noise.

Then the sequences of wavelet coefficients of the observation data is as follows :

$$y_j[n] = d_j[n] + w_j[n] \quad j=1, 2, \dots, J$$

Where  $d_j[n]$  and  $w_j[n]$  are the sequences of wavelet coefficients of the clock difference data and observation noise, respectively. The sequences  $\{w_j[n], n \in N\}$  are observation noises with variance  $\delta_w^2$ .

According to [8], the sequences  $d_j[n]$  are stationary processes for any scale. Hence, they can be approached on the basis of the AR model in the time-scale domain as

$$\begin{aligned} d_j[n] &= \sum_{i=1}^p \phi_i^j d_j[n-i] + e_j[n] \\ &= \phi^j \chi_j[n-1] + e_j[n] \end{aligned} \quad (18)$$

Where  $\{e_j[n], \dots, n \in N\}$  is a zero-mean model noise,  $p$  denotes the order of the AR model and the  $p$ -dimensional vectors are defined as

$$\begin{aligned} \chi_j[n-1] &= (d_j[n-1], \dots, d_j[n-p])^t \\ \phi^j &= (\phi_1^j, \phi_2^j, \dots, \phi_p^j) \end{aligned} \quad (19)$$

According to Yule-Walker equation

$$\begin{bmatrix} R_j(0) & R_j(1) & \dots & R_j(p-1) \\ R_j(1) & R_j(0) & \dots & R_j(p-2) \\ \dots & \dots & \dots & \dots \\ R_j(p-1) & R_j(p-2) & \dots & R_j(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \dots \\ a_n \end{bmatrix} = \begin{bmatrix} \delta_e^2 \\ 0 \\ \dots \\ 0 \end{bmatrix} \quad (20)$$

The optimal coefficients of the kalman filters are given by

$$\left\{ \begin{aligned} \phi_j &= h_j R_{x_j[n-1]}^{-1} \\ \delta_e^2 &= R_j(0) - h_j R_{x_j[n-1]}^{-1} h_j^t \\ h_j &= (R_j(1), R_j(2), \dots, R_j(p)) \\ R_{x_j[n-1]} &= \begin{bmatrix} R_j(0) & R_j(1) & \dots & R_j(p-1) \\ R_j(1) & R_j(0) & \dots & R_j(p-2) \\ \dots & \dots & \dots & \dots \\ R_j(p-1) & R_j(p-2) & \dots & R_j(0) \end{bmatrix} \end{aligned} \right. \quad (21)$$

Here,  $R_j(n), R_j(0)$  can be figure out by (14)~(17).

Based on the definition of  $X_j[n]$ , the state space

model

$$\begin{aligned} \chi_j[n] &= F_j \chi_j[n-1] + G e_j[n] \\ y_j[n] &= H_j \chi_j[n] + w_j[n] \end{aligned} \quad (22)$$

can be derived from (18), where

$$\begin{aligned} F_j &= \begin{bmatrix} \phi_1^j & \phi_2^j & \dots & \phi_p^j \\ 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \end{bmatrix} \\ G &= (1, 0, \dots, 0)^t \quad H = (1, 0, \dots, 0) \end{aligned} \quad (23)$$

Then, we can get the wavelet coefficients series  $d_j[n]$ ,

by use of kalman filters, we can get

$$\hat{\chi}_j[n] = (\hat{d}_j[n, n], \hat{d}_j[n, n-1], \dots, \hat{d}_j[n, n-p+1])^t \quad (24)$$

The kalman algorithm can be referred by [8].

## V. DATA PROCESSING

We process the single-channel and multi-channel raw common-view data in our experiment, respectively. During the pretreatment of single-channel common-view data, the CRL-NTSC in 2001(MJD(51912-52001)) is used as experiment data. As in the process of pretreatment, during the time interval of unlocked secondary planet or the intermission of common-view, the second-degree interpolation polynomial is not used to insert absent data. During the pretreatment of multi-channel common-view data, the NTSC-NICT in 2005 MJD(53367-53551) is used as experiment data, except satisfying strict common-view request, we delete the data of astral whose elevation less than 20 degree. In the following, we use two steps to demonstrate our data processing.

### V.I The Estimate of Exponent H

As we know, the fractal brown motion is a zero-mean Gaussian nonstationary random process, but its increment is a zero-mean stationary random process, so we figure out the exponent H of the fractal signal by transforming its increment with wavelet. Here, we make the pretreatment common-view data as the raw fractal signals, because their mean value is not

equal to zero, we make zero-mean process. (which cannot affect H parameter.) After some data process, we can further obtain its increment. That is  $G(t) = y_H(t+1) - y_H(t)$ . Then, we process them with wavelet transform according to Mallat algorithm. For the single-channel data and the multi-channel data, we all use five scales of Haar wavelet. Meanwhile, we figure out the H parameter use the method of what we have denoted. Figure 1(A) show the increment of the single-channel data of CRL-NTSC ( MJD(51912-52001)) in 2001. Whose H parameter is 0.6473. Figure 1(B) show the increment of the multi-channel data of NTSC-NICT (MJD(53367-53551)) in 2005, whose H parameter is 0.6726.

## V.II The Estimation of Clock Difference Data

We process two kinds of channel GPS common-view observation data, respectively. For the white noise  $\delta^2$  inserted in (14) and (16), we let it 1, just as the variance  $\delta_w^2$  of the observation noise in observation equation, we also let it 1. The other coefficients can be obtained by the optimal coefficients determined by (21). According to [3], the original value of  $d_j[n]$  is given by  $a_0^*[n] = y(n)$ . That is to say,  $y(1)$ ,  $y(2)$ ,  $y(3)$ ,  $y(4)$  are make as the original value of  $d_j[n]$ . But in our processing, because of considering correlation structure in our algorithm, the choice of original value of  $d_j[n]$  does not very important for the eventual results.

In this paper,  $P=4$ . For single-channel data, the filter bank is corresponding to 3 scales of Haar wavelet. For multi-channel, 6 scales of Haar wavelet are used. Finally, the results which only after an interval of five days, and at the time point of 389.5 second, the data can be reserved, are compared with circular T. The final results comparisons between CRL-NTSC ( MJD(51912-52001) ) and circular T, whose Root-Mean-Square is 5.20ns; the results comparisons between NTSC-NICT ( MJD(53367-53459) ) and circular T, whose Root-Mean-Square is 4.89ns. The series figures show the data before and after the filter banks.

## VI CONCLUSION

It must be remarked that the conclusions obtained in the paper are based on a hypotheses. We work out the exponent H of our common-view data based on this hypotheses. For  $0 < H < 1$ , we can continue to talk about the correlation structure of wavelet coefficients about our data. Then we can estimate the data what we need. This algorithm gives us a new method for estimate the time series data.

The other virtue of this algorithm lies in, except we let  $\delta^2 = 1$  and  $\delta_w^2 = 1$ , other coefficients can be obtained by theory equation, not by experience or transcendent method, which can reduce the man-made error by people themselves. Meanwhile, which give us a good approach to get optimal coefficients value. All of these help us to get better precision in the final results.

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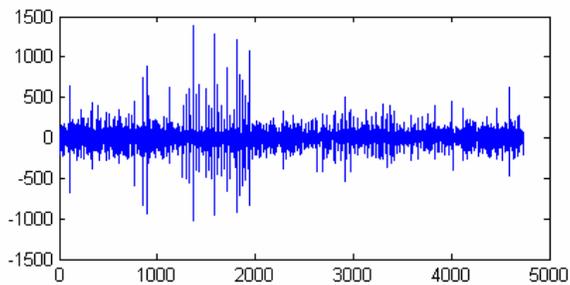


Figure 1(A) CRL-NTSC (MJD(51912-52001))  
the increment of single-channel clock difference data

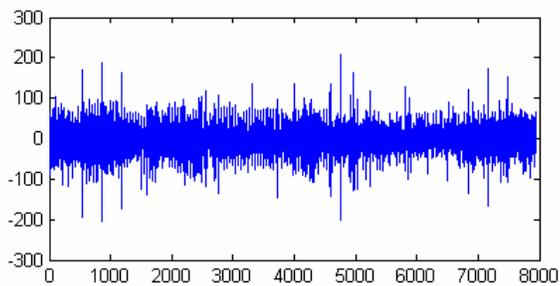


Figure 1(B) NTSC-NICT (MJD(53367-53551))  
the increment of multi-channel clock difference data

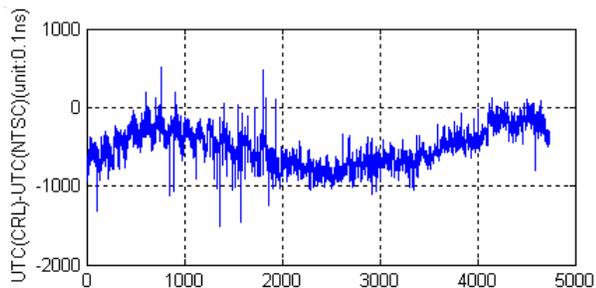


Figure 2(A) CRL-NTSC (MJD(51912-52001))

raw single-channel common-view data

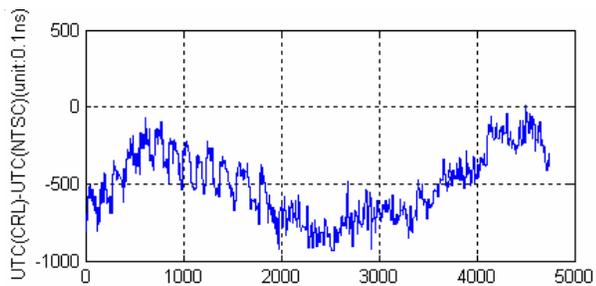


Figure 2(B) CRL-NTSC (MJD(51912-52001))

After the multikalman filters considering correlation structure

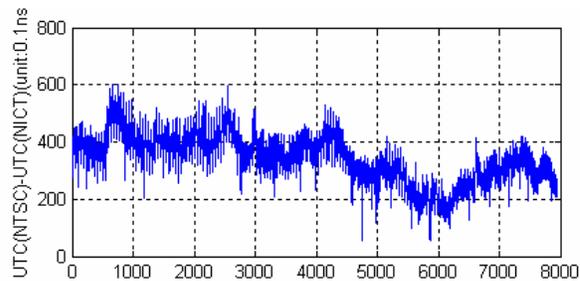


Figure 3(A) NTSC-NICT MJD(53367-53551)

Raw multi-channel common-view data

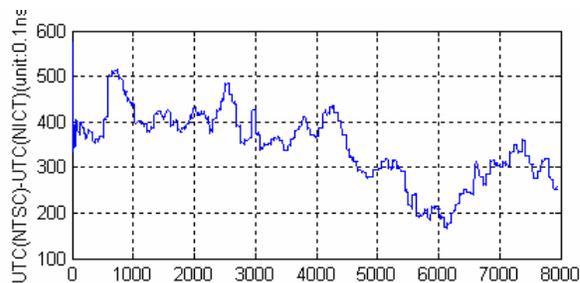


Figure 3(B) NTSC-NICT MJD(53367-53551)

After the multikalman filters considering correlation structure