APPLICATION OF HYDROGEN–MASER TECHNOLOGY TO THE SEARCH FOR GRAVITATIONAL RADIATION

John D. Anderson, John W. Armstrong, and Eunice L. Lau*
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109

Abstract

The Deep Space Network (DSN) is currently able to track distant spacecraft with a worldwide network of 70 m stations located near Goldstone, California; Tidbinbilla, Australia and Madrid, Spain. Hydrogen–maser timekeeping at all three sites makes possible a search for gravitational radiation at useful levels of sensitivity. We report on 10.5 days of data from the December 1988 opportunity with the Pioneer 10 spacecraft at a distance of 44.2 astronomical units (22052 s). A microwave link near 2.3 GHz was established with the spacecraft using one station of the DSN on the uplink and another on the downlink. The utilization of hydrogen masers at each station, plus the spacecraft transponder, effectively provided a one–arm interferometer for detecting gravitational radiation with a phase–coherent path length of 12.25 hr.

Starting with integrated cycle count of the Doppler shift as raw data, we sequentially differenced the data at a sample interval of 60 s and thereby generated a Doppler frequency record. Next, we removed trends caused by the relative motions of the spacecraft and the DSN stations in inertial space. The resulting residual frequency record was dominated by refraction of the 2.3 GHz signal in the Earth’s atmosphere and ionosphere. By applying a regression analysis to a stratified atmospheric model, we reduced these refraction effects by more than an order of magnitude.

We show plots of the detrended Doppler residuals at the 60 second sample interval. The overall RMS residual is on the order of 2 mHz (10^{-12} in fractional frequency), but the major contributor to the noise at this level is the high–frequency component caused by the poor signal to noise ratio in the received signal from the distant Pioneer spacecraft. We illustrate this property by displaying the power spectral density of the reduced data. As expected, the high–frequency end of the spectrum is typical of thermal noise in the microwave receiver. On the other hand, the low frequency end is dominated by refraction of the signal by interplanetary plasma. The lowest noise is achieved for a period range from 200 s to 2000 s where the advantage of hydrogen–maser timekeeping is most apparent. In this region, the local mean amplitude of the noise is on the order of 10^{-14} in fractional frequency. This implies that any sinusoidal gravitational waves in the bandwidth are limited to a strain amplitude of 4 \times 10^{-14}.

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INTRODUCTION

From 1 December to 10 December 1988 the Deep Space Network (DSN), using three 70 m stations in California, Australia, and Spain respectively, provided nearly continuous radio tracking of the Pioneer 10 spacecraft at a single S-band frequency near 2.3 GHz. Each December Pioneer 10 is nearest solar opposition, an optimum location for minimizing the effects of the solar wind on the telecommunication link. Being a spinning spacecraft with a minimum of nongravitational forces, at a distance from Earth of 44.2 astronomical units (6 hr 7.5 min), it is uniquely suited for a search for gravitational radiation. In principle, gravitational waves (GW) crossing the solar system can be detected by a one-arm interferometer consisting of a phase-coherent radio link between Earth and a distant spacecraft. Such a link can be established with the Pioneer spacecraft by transmitting an S-band signal from one 70 m station and receiving the transponded signal at another station separated by intercontinental distances. Coherency is maintained by means of hydrogen masers at both stations.

As a GW detector, the one-arm interferometer at the distance of Pioneer 10 in 1988 has an effective pathlength of 12.25 hr, the round-trip light time (RTLT) of the radio signal. Ideally it has a broad-band response and a sensitivity of parts in $10^{15}$ in the very-low-frequency (VLF) region of the GW spectrum from about $7 \times 10^{-5}$ Hz (period $\approx$ RTLT/3) to $5 \times 10^{-5}$ Hz, where the upper frequency limit is set by thermal noise in the microwave receiver, and the lower limit by the response function to gravitational radiation. But in fact the inherent stability provided by the hydrogen masers (parts in $10^{15}$) does not determine the sensitivity limit for the radio link to the Pioneer spacecraft. Instead the useful level of sensitivity is limited by the propagation of the radio signal through both neutral and ionized media. Nevertheless by computing the power spectral density and Allan variance of the reduced data, we can show that the current experiments approach a level of sensitivity to spatial strain of $10^{-14}$ in a $10^{-6}$ Hz bandwidth over a limited GW band of $3 \times 10^{-3}$ to $5 \times 10^{-4}$ Hz. Even better sensitivities, by at least a factor of 10, are expected for future missions in the mid to late 1990’s, at which time the experiments may approach the limiting accuracy of the DSN’s frequency and timing system.

BACKGROUND

The idea of using the Doppler tracking of distant spacecraft to detect gravitational radiation originated with A. J. Anderson[11] about 20 years ago. In that early period Anderson looked into the feasibility of the technique[8] and concluded that it was worth pursuing. In the meantime, the theory of the interaction of gravitational radiation with a Doppler link was developed in 1975 by Estabrook and Wahlquist[9]. Over the next few years the theory was developed further[4,5,6,7] and over the last few years some experimental results have appeared[8,9,10,11]. No detection has been reported, but some interesting limits have been set on a stochastic cosmic background of radiation[8,10], burst sources[8,11], and sinusoidal sources[9,11]. The December 1988 opportunity with the Pioneer 10 spacecraft resulted in the best set of GW Doppler data to date. The distance to the spacecraft ranged from 22051 s on 1 December 1988 to 22087 s on 10 December 1988.

DATA

The raw Doppler data, delivered to us by the DSN on magnetic tape, consist of records of cumulative cycle count at a sample interval of 1 s. Before these data can be filtered for GW signals, some data reduction is required. First we use software of the Navigation Subsystem (NAV) at JPL to convert cycle count (phase) to a finite differenced record of frequency at a sample interval of 60 s. Because
the spatial strain of gravitational radiation is directly proportional to the Doppler frequency shift\cite{3}, it is this differenced record that is most useful for GW analysis. Next, again using NAV software, we remove the Doppler motion effects from the data with the best Pioneer 10 orbit available, and with the best available models for the station locations, Earth rotation, and Earth polar motion. These geodynamic models are provided to the users of NAV software by the DSN as part of their multimission support.

During the initial data reduction with NAV software, we also introduce a bias into the frequency data to account for the rotation of the Pioneer spacecraft and its parabolic high-gain antenna. The rotating antenna introduces a Doppler shift of one cycle per rotation period. For the 1988 data, the spin period, as determined by the Pioneer Project from onboard photopolarimeter data, varies approximately linearly from 13.365123 s on 1 December to 13.366524 s on 10 December. Because the spacecraft spin also affects the physical motion of the antenna feed in space, a time-varying periodicity appears in the reduced data. However this small signal can easily be identified and removed, if necessary. In fact, because we know such a signal is present, it can be used to advantage in testing filtering algorithms for unknown small signals whose periods vary slowly with time. Such signals, with period decreasing with time, are expected from coalescing binary stars in the final stages of their radiation of gravitational waves\cite{12}.

After we have reduced the data with NAV software, the dominant colored noise that remains is generated by refraction of the radio beam in the Earth's atmosphere and ionosphere. Though the DSN provides a seasonal tropospheric model to NAV, we find that over just ten days of data we can remove more of the effects of atmospheric refraction by applying a regression analysis to a parameterized stratified model of the atmosphere. Stratification assures that the stationary component of both the troposphere and ionosphere are accounted for. The resulting reduced data record, which is the starting point for a search for gravitational waves, is shown in Figure 1. The residual Doppler shift is plotted against time from 1 December, 1988. The average of the 60 s data at an averaging interval of 300 s is shown in Figure 2. The obvious reduction in data noise in Figure 2 results from an effective integration of the received signal beyond the point where high-frequency thermal noise dominates.

**LIMITING NOISE SOURCES**

The power spectrum of the reduced data of Figure 1 is shown in Figure 3, and the corresponding Allan variance is plotted in Figure 4. It is apparent that at high Fourier frequencies the noise is dominated by $f^2$ white phase noise. However the magnitude of the Allan variance is much too big to be attributable to the hydrogen masers, and it probably exceeds reasonable limits on the performance of the 70 meter stations. The thermal noise extrapolates to $5.3 \times 10^{-14}$ at 1000 s. We suggest that this rather high level of thermal noise is caused by a very weak received signal from Pioneer 10 transmitting at 8 W into a 3 m parabolic antenna at a distance of 44.2 AU.

From the viewpoint of the search for gravitational radiation, the most serious limiting noise source is not the thermal component, but instead the $1/f$ flicker-frequency fluctuations. This component dominates beginning at about a 300 s integration time and continues indefinitely for longer integration times. The Allan variance is constant in this region with a limiting value of $2.8 \times 10^{-13}$. The source of the noise can be traced to random fluctuations in the solar wind\cite{11}. The effect of these plasma fluctuations on the Doppler data is proportional to the inverse square of the frequency of the radio transmission. Fortunately future gravitational wave searches will be conducted at higher frequency.

Two recent missions, Galileo to Jupiter and Magellan to Venus, have the capability of transmitting in
the X band on both the up and down links. The higher frequency, by a factor of 11/3 over previous S-band experiments, would suggest an Allan variance of $2 \times 10^{-14}$ for the equivalent plasma noise experienced by Pioneer 10 in 1988. Indeed early examination of Doppler data from Magellan at X band suggests a noise floor at this level. Being close to the Earth, Magellan has been tracked by the new 34 m High Efficiency (HEF) stations of the DSN. The Magellan data demonstrate that the overall HEF tracking system, including the hydrogen masers, is at least as good as $2 \times 10^{-14}$. On the other hand, if the noise in the Magellan data is dominated by plasma fluctuations near Earth, then the HEF system may perform even better, perhaps to parts in $10^{-15}$. Future X-band tracking of Galileo, after deployment of its high-gain antenna in May 1991, may reach this improved level of sensitivity, particularly in a more benign plasma environment.

CONCLUSIONS

We have shown that current GW Doppler experiments at S band are not limited by hydrogen-maser stability, nor even by the overall frequency and timing system of the DSN. Instead noise introduced on the tracking link by random fluctuations in the solar wind limit current experiments. In the future when similar experiments can be performed in the X band, using the Galileo orbiter for example, the noise from the solar wind may be reduced to a level where the limiting accuracy of the frequency and timing system will be important.

Though limited by plasma noise, the current GW search with the Pioneer 10 spacecraft should continue for as long as coherent tracking is possible. The Galileo spacecraft will not venture beyond the orbit of Jupiter, so its phase-coherent pathlength (maximum RTLT of 8.5 AU) will severely limit its response to GW radiation in regions of the spectrum where Pioneer 10 is particularly sensitive (RTLT $\approx$ 100 AU). The Pioneer 10/11 spacecraft are unique in their potential for detecting and characterizing broad-band gravitational radiation bursts with duration of a few hours.

By accumulating several days of phase-coherent data from Pioneer 10 near solar opposition, a limiting sensitivity of $2.8 \times 10^{-13}$ in the Allan variance is not as bad as one might suppose. In Figure 5 we show the limiting sensitivity to spatial strain for sinusoidal signals within about 6800 independent spectral bins. These amplitudes, obtained by taking the square root of the power multiplied by the appropriate resolution bandwidth in Hz, represents our best estimate of the amplitudes of sinusoidal signals in the reduced data record of Figure 1. The RMS sensitivity to strain amplitude is about $1 \times 10^{-14}$. The corresponding 90% confidence limit for a real signal in these exponentially distributed spectral estimates is $4.1\sigma = 4 \times 10^{-14}$. A discussion of the relationship between the upper limit on the strain amplitude determined from the Doppler data and the corresponding upper limits on the GW excitation can be found elsewhere[11].

Finally we point out that clock technology probably has important applications to other space experiments designed to detect gravitational radiation. In particular, spaceborne clocks could be used to establish multilink radio tracking between the Earth and one or more spacecraft, or very large microwave GW antennas utilizing several freeflying spacecraft could be entirely spaceborne[13]. Though we have not addressed these issues here, we encourage the support of studies that do.

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REFERENCES


Figure 1: Reduced Data at a Doppler Integration Time of 60 Seconds

Figure 2: Reduced Data at a Doppler Integration Time of 300 Seconds
Figure 3: Power spectrum of frequency residuals.

Figure 4: Allan variance of frequency residuals.
Figure 5: Equivalent sinusoidal strain amplitude versus Fourier period for the frequency residuals.
QUESTIONS AND ANSWERS

UNIDENTIFIED QUESTIONER: The gravitational radiation that you are looking for, is that from supernovas, black holes—what in particular are you looking for?

MR. ANDERSON: That is a good question. There are some predictions by the theorists about what we might see at these frequencies. The black hole is a possibility, the coalescing binary black hole is something that we might see, supernovas are at the higher frequencies, in the kilohertz region, while, as you see here, we are in the lower frequency range from 300 seconds out to thousands of seconds. The resonant bar detectors on the earth are better for detecting the supernova collapse that occur on the order of milliseconds. We would be looking for cataclysmic events, either galactic or extra-galactic. One of the things that we are considering is coalescing binaries. There would be a sinusoid of gravity radiation with the frequency of the sinusoid slowly changing as the binaries gradually coalesce. Those are the kinds of sources that we are examining.

SAME QUESTIONER: Can you derive vectors as to where the radiation is coming from?

MR. ANDERSON: Yes, in that cartoon there is a directional dependence on the shift. It affects both where the third pulse occurs, and the amplitudes of the three pulses because they have to sum to zero. That is, after the wave passes, there is no memory left in the Doppler link, you have to observe it as it passes. You might think of this as a single arm interferometer detecting a gravitational wave. You can make a determination of this angle, Θ, with some precision when you make a detection, which will determine a small circle on the sky. You can’t get an unambiguous direction from a single arm interferometer. You need multi-arm interferometers for that. We might think of using space-borne Hydrogen Masers in the future for multi-arm interferometers, but for now we will be just able to describe the small circle on the sky.