EVALUATION OF MITREX MODEM TRANSMIT AND RECEIVE DELAY INSTABILITY

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Abstract

The IEN time and frequency laboratory has developed an automated TWSTFT station to join the experimenters network on INTELSAT satellite already operative. The hardware and software that were first checked in April 1999 when the IEN station participated in the international measuring sessions using the OCA code and time slot are still under test. The chance of having three MITREX modems available at the laboratory, allowed us to measure transmitting and receiving delays by suitable cross-connections between the modems. In the measurement setup a common reference frequency has been applied to the modems and all the possible combinations of (Tx + Rx) time interval measurements have been performed. Unfortunately, the measurement scheme doesn't allow the estimation of the values of the separate transmitting and receiving delays but, under suitable assumptions, it may be shown that the variances and covariances of the individual delays may be estimated from the variances and covariances of the measured quantities. This estimation requires a noise decoupling technique similar to the known N-cornered-hat method used to evaluate the noises of the individual clocks in comparison measures. This can give an insight of the instabilities of the modem channels both in the short term and in the long term as one of the instability contribution in the calculation of the TWSTT uncertainty budget.

INTRODUCTION

A Two-Way Satellite Time and Frequency Transfer (TWSTFT) station has been put in operation during 1999 at the IEN time and frequency laboratory. This equipment will permit the IEN to join the other experimenters on the INTELSAT 706 TWSTFT network during the scheduled measurement sessions. The Two-Way is the most accurate and precise method of comparing two remote clocks because of the reciprocity of the transmit and receive path delays. The accuracy obtained is dependent on the accuracy of the determination of the non-reciprocal part of these delays. As seen in other papers [1] the non-reciprocity in the earth stations is the limiting factor for absolute time transfer. The earth station delays depend on many factors, the transmitting and receiving VSAT station, the length and quality of signal cables, the modem used for the transmission (MITREX, SATRE), and so on. During the setting up of the TWSTFT station, we had the opportunity of having three MITREX modems at the same time, so we took the occasion of evaluating the stability of the transmitting and receiving delays by suitable cross-connection of the three modems.
In other papers [2] are described ways to measure the value of the transmit delay of the modem using an oscilloscope, but in this paper we discuss a method that permits to use the same measurement setup used during the normal TWSTFT session.

**MEASUREMENT SETUP**

The measurement setup permits to determine the transmit plus receive delay of all the possible combination of the three MITREX modems connections, nine measures in all. The three modems have been connected to the same 10 MHz reference frequency by a distribution amplifier and have been synchronized to UTC(IEN) at the beginning of the measurement period. The measures have been performed during four weeks from August 30 to November 23 1999 (MJD 51419 to 51444) at the IEN Time and Frequency laboratory, where the temperature is controlled within ±1°C, as visible in Fig. 1. This will permit to decouple the influence of temperature during the computation of results, as shown below.

As can be seen in Fig. 2, nine different possible cross-connections of the three MITREX modems have been implemented on the 70 MHz signal using the TX, RX, and 1PPS-TX, 1PPS-RX connectors. To avoid different delay contributions, the same cables have been used for each measurement connection. The time-interval counter used was a Stanford Research System model SR620, while the three MITREX modems were:

<table>
<thead>
<tr>
<th>Modem a</th>
<th>MITREX 2500a PC</th>
<th>IEN (Istituto Eletrotecnico Nazionale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modem b</td>
<td>MITREX 2500a PC</td>
<td>INS (Institute of Navigation Stuttgart)</td>
</tr>
<tr>
<td>Modem c</td>
<td>MITREX 2500a ISCTI</td>
<td>ISCTI (Istituto Superiore delle Comunicazioni e Tecnologie dell’Informazione)</td>
</tr>
</tbody>
</table>

Measured data have been collected every second by a PC program using the IEEE488 communication interface of the SR620 counter. The counter used the same 10 MHz reference clock of the three MITREX modems as time base provided by the IEN master clock. Counter instability was estimated at level of 20ps, as visible in Fig. 3. Each configuration was measured during a period of 2 minutes (like in a normal TWSTFT measurement session) and every time the cross-connection was manually changed. As visible in Fig. 4, the three modems were installed in a way to avoid overheating and they were never turned off during all the four-week period of the measurement.

**MATHEMATICAL DATA ANALYSIS**

Through the described measurement scheme it is thus possible to measure 9 different quantities corresponding to the different connections between each modem transmitting part versus the receiving part of another one. Unfortunately, such 9 measures are not sufficient to estimate the quantities of interest, i.e. the values of the separate transmitting and receiving delays. Nevertheless, it can be of interest to estimate the instability of the modem individual receiving or transmitting delays and that is done by applying the “decoupling noise” technique typically used in the known “N-cornered-hat method” In this case too things are not that perfect because we always wish to estimate more quantities than what we can observe and, therefore, some additional assumptions are necessary. This work has to be considered a first and provisional attempt to face this problem, because different tests are still necessary to achieve fully confident results. Nevertheless, the provisional estimates here obtained are encouraging on the possibility of successfully using this method.
Let’s introduce some useful notations. Let \( Y \) be the vector containing the 9 possible measures as:

\[
Y = \begin{bmatrix}
T_a + R_a \\
T_a + R_b \\
T_a + R_c \\
T_b + R_a \\
T_b + R_b \\
T_b + R_c \\
T_c + R_a \\
T_c + R_b \\
T_c + R_c
\end{bmatrix}
\]

and \( X \) the vector of the 6 unknown quantities (the change in order of “\(a,b,c\)” was dictated by the subsequent development):

\[
X = \begin{bmatrix}
T_a \\
T_b \\
T_c \\
R_b \\
R_c \\
R_a
\end{bmatrix}
\]

It can easily be written that unknowns and measures are related by a “design matrix” \( M \) as:

\[
Y = M \cdot X
\]

(3)

with \( M \) appropriately filled with 0 and 1. The covariance matrix of the unknowns can be indicated by \( \Sigma_X \) and it contains variances and covariances of all the unknowns. The variance \( \sigma_i^2 \) of the \( i \)-th unknown is accommodated along the main diagonal while, outside the diagonal, the covariance \( \sigma_{ij} \) between the different couple \((i,j)\) of unknowns is placed. Any covariance matrix is symmetric.

\[
\Sigma_X = \begin{bmatrix}
\sigma_{T_a}^2 & \sigma_{T_a T_b} & \sigma_{T_a T_c} & \sigma_{T_a R_b} & \sigma_{T_a R_c} & \sigma_{T_a R_a} \\
\sigma_{T_b}^2 & \sigma_{T_b T_b} & \sigma_{T_b T_c} & \sigma_{T_b R_b} & \sigma_{T_b R_c} & \sigma_{T_b R_a} \\
\sigma_{T_c}^2 & \sigma_{T_c T_c} & \sigma_{T_c R_b} & \sigma_{T_c R_c} & \sigma_{T_c R_a} \\
\sigma_{R_b}^2 & \sigma_{R_b R_b} & \sigma_{R_b R_c} & \sigma_{R_b R_a} \\
\sigma_{R_c}^2 & \sigma_{R_c R_c} & \sigma_{R_c R_a} \\
\sigma_{R_a}^2 & \sigma_{R_a R_a}
\end{bmatrix}
\]

(4)

Analogously the covariance matrix \( \Sigma_Y \) of the measures can be written. It is known that when the vectors \( X \) and \( Y \) are related as in (3), also the relationship between corresponding covariance matrices is known and written as:

\[
\Sigma_Y = M \cdot \Sigma_X \cdot M^T
\]

(5)

where superscript \( T \) indicates transposition.
Even if the measure number is larger than the number of unknowns, these measures \( Y \) are not sufficient to estimate \( X \) and it can be seen from the fact that the rank of \( M \) (number of independent columns) is equal to 5, therefore less than the number of unknowns. Only 5 independent unknowns could be estimated from this set of measures. The same happens for the covariance matrices. \( \Sigma_Y \) could be estimated by evaluating the variances and covariances of measurement series, but the knowledge of \( \Sigma_Y \) is not sufficient to completely determine \( \Sigma_X \) that contains, as in (4), the instability quantities of interest. For a detailed definition of variances and covariances of measurement series, see, for example, [4, 5, 6]. To overcome this limitation, it was supposed that some of the matrix elements in (4) are known. In particular, it was supposed that some covariance terms denoting correlation should be set to zero, as they concern the correlation between variables that can be supposed independent. It was considered that the instability in the receiving delays of couples of different modems, as well as the receiving and transmitting instabilities of different modems, should be independent. It means:

\[
\sigma_{Ti}R_j = \sigma_{Ri}R_j = 0 \quad \text{for} \quad i \neq j
\]

(6)

On the other hand, the transmitting instabilities of different modems could be correlated by the fact that the same clock is used to trigger the transmission (\( \sigma_{T_iT_j} \neq 0 \)) and the same was assumed about the receiving and transmitting instabilities of the same modem (\( \sigma_{TiR_i} \neq 0 \)). These assumptions seem reasonable from the knowledge of modem operation mode, but nevertheless, from a mathematical as well as physical points of view, they are very strong. For this fact the model has to be considered a very first estimation attempt. Unfortunately if some assumptions are relaxed, it means an important entanglement of the mathematical treatment that deserve some further investigation. With the proposed assumption, the covariance matrix (4) simplifies to:

\[
\Sigma = \begin{bmatrix}
\sigma^2_{T_a} & \sigma_{T_aT_b} & \sigma_{T_aT_c} & 0 & 0 & \sigma_{T_aR_a} \\
\sigma_{T_aT_b} & \sigma^2_{T_b} & \sigma_{T_bT_c} & \sigma_{T_bR_a} & 0 & 0 \\
\sigma_{T_aT_c} & \sigma_{T_bT_c} & \sigma^2_{T_c} & 0 & \sigma_{T_cR_a} & 0 \\
0 & \sigma_{T_bR_a} & 0 & \sigma^2_{R_a} & 0 & 0 \\
0 & 0 & \sigma_{T_cR_a} & 0 & \sigma^2_{R_c} & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2_{R_i}
\end{bmatrix}
\]

(7)

The number of unknown elements in (7) are now reduced to 12. The quantities of main interest are on the main diagonal, in particular:

\( \sigma^2_{Ti} \) is the variance, therefore the instability, of the transmitting delay of the \( i \)-th modem

\( \sigma^2_{R_i} \) is the variance, therefore the instability, of the receiving delay of the \( i \)-th modem
By the aid of the relationship (5) and the simplifying assumptions (6), the matrix (7) can now be estimated. This was done with the measures $Y$ taken over different days and averaged over different integration time $\tau$. A preliminary estimation of the separate instabilities of the transmitting or receiving delays was, thus, possible.

RESULTS

In Fig. 5 the different measures as indicated in $Y$ are reported. Each point represents the averaged value of the 120 measures spaced by 1 s. This measurement was repeated on different days at the same hour. Not every week contains the same number of measures; on the whole, 9 day of measures are available. All the devices were never switched off during the entire 4-week measurement period. It can be noticed that the reported values can change from day to day by hundreds of ps. This gives a first idea of the stability from day to day of the modem delays.

By applying the decoupling procedure described in the previous section, the instabilities of the separate transmitting and receiving channels were estimated. Firstly, for each measurement day, the instability over 1 s was estimated by taking the classical variance of the 120 measures spaced by 1 s and the procedure was repeated for each day of available measures. The classical variances and covariances of the measurement series at $\tau = 1$ s were evaluated and arranged in the corresponding matrix $\Sigma_Y$; then the covariance matrix $\Sigma_X$ of separated delays was estimated for each measure day. In particular, the deviations $\sigma_{T_i}(1s)$ and $\sigma_{R_i}(1s)$ of the $i$-th modem transmitting and receiving delays were evaluated on each measurement day. The results are reported in Fig. 6. For each day of available measures the procedure was repeated and a different estimate of the separated delay instabilities is reported. Even if the mathematical treatment relies on simplifying assumptions and results are to be considered only preliminary and approximate, they show some very interesting features. For example, the instability of the transmitting delays of the 3 modems are very similar; their deviations are in the region of about 150 ps and can change from day to day by about 50 ps. It seems that in the transmitting parts the 3 devices do not have a different behavior. The instability of the receiving delay of modem “a” and “c” are also similar, but different from the instability of the receiving part of the modem “b”. Also, from an inspection of the measurement results reported in Fig. 5, it was suspected that the modem “b” was more noisy, probably due to the fact that the input attenuator of this device was defective. Apart from modem “b”, it could be found that the receiving delay is more stable than the transmitting delay. A possible explanation could be found in the presence, only in the transmitting chain, of a 70 MHz band pass filter.

As a second step, the measures spaced of 1 s were averaged over $\tau = 2, 5, 10,$ and 20 s inside each daily bin of 120 measures. Therefore, new sequence of averaged measures were obtained for each day and the decoupling procedure was repeated for every different averaging time and reported in Fig. 7. Actually, for any measurement day, a different estimate of the instabilities was available and the values reported in Fig. 7 are the mean values over the entire measurement period. The classical deviation for $\tau = 2, 5, 10,$ and 20 s and for each transmitting and receiving part was estimated. Fig. 7 reports also the slope $1/\sqrt{\tau}$ that corresponds to the slope of the classical deviation in case of white phase noise. It can be seen that, as in the previous figure, the instabilities of the transmitting parts are very similar to each other and a bit more noisy than the instabilities of the receiving parts, apart from the instability of the receiving part of modem “b”, which is confirmed to be the more noisy. Moreover, the slope of the deviation is in any case not that different from the slope $1/\sqrt{\tau}$.
corresponding to a white phase noise, thus indicating that both the transmitting or receiving instabilities can safely be considered as white phase noise. Unfortunately, the analysis was only possible up to $\tau = 20$ s and, therefore, this consideration regards only the very short-term instability, but this is of interest because it is on the very short period (120 s) that the TWSTT is carried out.

**CONCLUSIONS**

The instabilities of the transmitting and receiving part of the 3 modems was estimated, by means of a “decoupling noise” technique that tries to estimate the covariance matrix of individual instabilities from the comparison measurement. Unfortunately, the possible comparison measurements are not sufficient to estimate all the instabilities of the separate parts of the modems; therefore, some assumption on the possible correlations were necessary. This is only a very preliminary evaluation and setting up of the model, because these assumptions should be further tested and, if possible, relaxed. To this aim, the mathematical treatment needs some more investigation. Nevertheless, some interesting results were obtained giving an idea of the short-term instabilities of the receiving and transmitting modem delays and also on the repeatability of the values of these delays from day to day. The necessity to measure at each session the modem delays to achieve the best uncertainty associated to the Two-Way technique is consequently confirmed.

**REFERENCES**


Fig. 1 - IEN Time and Frequency Laboratory temperature behavior.

Fig. 2 - Block diagram of the measurement setup.
Fig. 3 - Time-interval counter instability.

Fig. 4 - Measurement setup at the IEN Time and Frequency laboratory.
Fig. 5 - Standard deviation of daily data.

Fig. 6 - Standard deviation of separated modem delays.
Phase deviation $\sigma(\tau)$

Fig. 7 - Phase deviation of separated delays.